



$$8 \quad \begin{array}{lll} \mathbf{a} = \frac{1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{1}{5}\sqrt{5} & \mathbf{b} = \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2}{3}\sqrt{3} & \mathbf{c} = \frac{1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{1}{4}\sqrt{2} \\ \mathbf{d} = \frac{14}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = 2\sqrt{7} & \mathbf{e} = \frac{3\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \sqrt{6} & \mathbf{f} = \frac{\sqrt{5}}{\sqrt{3}\sqrt{5}} = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{1}{3}\sqrt{3} \\ \mathbf{g} = \frac{1}{3\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{1}{21}\sqrt{7} & \mathbf{h} = \frac{12}{6\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \sqrt{2} & \mathbf{i} = \frac{1}{4\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{1}{20}\sqrt{5} \\ \mathbf{j} = \frac{3}{6\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{1}{12}\sqrt{6} & \mathbf{k} = \frac{8\sqrt{5}}{9\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{4}{9}\sqrt{10} & \mathbf{l} = \frac{15\sqrt{7}}{6\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{5}{6}\sqrt{21} \end{array}$$

$$9 \quad \begin{array}{lll} \mathbf{a} = 2\sqrt{2} + \frac{6}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} & \mathbf{b} = 4\sqrt{3} - \frac{10}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} & \mathbf{c} = \frac{6-2\sqrt{2}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\ = 2\sqrt{2} + 3\sqrt{2} & = 4\sqrt{3} - \frac{10}{3}\sqrt{3} & = \frac{6\sqrt{2}-4}{2} \\ = 5\sqrt{2} & = \frac{2}{3}\sqrt{3} & = 3\sqrt{2} - 2 \\ \mathbf{d} = \frac{3\sqrt{5}-5}{2\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} & \mathbf{e} = \frac{1}{3\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} + \frac{1}{4\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} & \mathbf{f} = \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} - \frac{\sqrt{2}\sqrt{3}}{6\sqrt{2}} \\ = \frac{15-5\sqrt{5}}{10} & = \frac{1}{6}\sqrt{2} + \frac{1}{8}\sqrt{2} & = \frac{2}{3}\sqrt{3} - \frac{1}{6}\sqrt{3} \\ = \frac{1}{2}(3 - \sqrt{5}) & = \frac{7}{24}\sqrt{2} & = \frac{1}{2}\sqrt{3} \end{array}$$

$$10 \quad \begin{array}{ll} \mathbf{a} \quad x^2 + 4x = 4x + 32 & \mathbf{b} \quad x - 4\sqrt{3} = 2\sqrt{3} - 2x \\ x^2 = 32 & 3x = 6\sqrt{3} \\ x = \pm\sqrt{32} & x = 2\sqrt{3} \\ x = \pm 4\sqrt{2} & \\ \mathbf{c} \quad 3\sqrt{2}x - 4 = 2\sqrt{2} & \mathbf{d} \quad \sqrt{5}x + 2 = 2\sqrt{5}(x-1) \\ 6x - 4\sqrt{2} = 4 & 5x + 2\sqrt{5} = 10(x-1) \\ 6x = 4 + 4\sqrt{2} & 5x = 10 + 2\sqrt{5} \\ x = \frac{2}{3}(1 + \sqrt{2}) & x = 2 + \frac{2}{5}\sqrt{5} \end{array}$$

$$11 \quad \begin{array}{l} \mathbf{a} = 4 - (\sqrt{3})^2 = 4 - 3 = 1 \\ \mathbf{b} = \frac{2}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{2(2+\sqrt{3})}{1} = 4 + 2\sqrt{3} \end{array}$$

$$12 \quad \begin{array}{l} \mathbf{a} = \frac{1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{\sqrt{2}-1}{2-1} = \sqrt{2}-1 \\ \mathbf{b} = \frac{4}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{4(\sqrt{3}+1)}{3-1} = 2(\sqrt{3}+1) \\ \mathbf{c} = \frac{1}{\sqrt{6}-2} \times \frac{\sqrt{6}+2}{\sqrt{6}+2} = \frac{\sqrt{6}+2}{6-4} = \frac{1}{2}(\sqrt{6}+2) \text{ or } \frac{1}{2}\sqrt{6}+1 \\ \mathbf{d} = \frac{3}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{3(2-\sqrt{3})}{4-3} = 3(2-\sqrt{3}) \\ \mathbf{e} = \frac{1}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}} = \frac{2-\sqrt{5}}{4-5} = \sqrt{5}-2 \end{array}$$

$$f = \frac{\sqrt{2}}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{\sqrt{2}(\sqrt{2}+1)}{2-1} = 2 + \sqrt{2}$$

$$g = \frac{6}{\sqrt{7}+3} \times \frac{\sqrt{7}-3}{\sqrt{7}-3} = \frac{6(\sqrt{7}-3)}{7-9} = 3(3-\sqrt{7})$$

$$h = \frac{1}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}} = \frac{3-2\sqrt{2}}{9-8} = 3-2\sqrt{2}$$

$$i = \frac{1}{4-2\sqrt{3}} \times \frac{4+2\sqrt{3}}{4+2\sqrt{3}} = \frac{4+2\sqrt{3}}{16-12} = \frac{1}{2}(2+\sqrt{3}) \text{ or } 1 + \frac{1}{2}\sqrt{3}$$

$$j = \frac{3}{3\sqrt{2}+4} \times \frac{3\sqrt{2}-4}{3\sqrt{2}-4} = \frac{3(3\sqrt{2}-4)}{18-16} = \frac{3}{2}(3\sqrt{2}-4) \text{ or } \frac{9}{2}\sqrt{2}-6$$

$$k = \frac{2\sqrt{3}}{7-4\sqrt{3}} \times \frac{7+4\sqrt{3}}{7+4\sqrt{3}} = \frac{2\sqrt{3}(7+4\sqrt{3})}{49-48} = 2(7\sqrt{3}+12)$$

$$l = \frac{6}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \frac{6(\sqrt{5}+\sqrt{3})}{5-3} = 3(\sqrt{5}+\sqrt{3})$$

$$13 \quad 3x = \sqrt{5}x + 2\sqrt{5}$$

$$x(3 - \sqrt{5}) = 2\sqrt{5}$$

$$x = \frac{2\sqrt{5}}{3-\sqrt{5}} = \frac{2\sqrt{5}}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}} = \frac{2\sqrt{5}(3+\sqrt{5})}{9-5}$$

$$x = \frac{6\sqrt{5}+10}{4} = \frac{3}{2} + \frac{3}{2}\sqrt{5}$$

$$14 \quad l = \frac{6}{3\sqrt{2}-3} = \frac{6}{3\sqrt{2}-3} \times \frac{3\sqrt{2}+3}{3\sqrt{2}+3} = \frac{6(3\sqrt{2}+3)}{18-9}$$

$$l = \frac{18(\sqrt{2}+1)}{9} = 2\sqrt{2} + 2$$

$$15 \quad a = \frac{\sqrt{2}}{\sqrt{2}+\sqrt{6}} \times \frac{\sqrt{2}-\sqrt{6}}{\sqrt{2}-\sqrt{6}} = \frac{\sqrt{2}(\sqrt{2}-\sqrt{6})}{2-6} = -\frac{1}{4}(2-2\sqrt{3}) = \frac{1}{2}(\sqrt{3}-1)$$

$$b = \frac{1+\sqrt{3}}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{(1+\sqrt{3})(2-\sqrt{3})}{4-3} = 2 - \sqrt{3} + 2\sqrt{3} - 3 = \sqrt{3} - 1$$

$$c = \frac{1+\sqrt{10}}{\sqrt{10}-3} \times \frac{\sqrt{10}+3}{\sqrt{10}+3} = \frac{(1+\sqrt{10})(\sqrt{10}+3)}{10-9} = \sqrt{10} + 3 + 10 + 3\sqrt{10} = 13 + 4\sqrt{10}$$

$$d = \frac{3-\sqrt{2}}{4+3\sqrt{2}} \times \frac{4-3\sqrt{2}}{4-3\sqrt{2}} = \frac{(3-\sqrt{2})(4-3\sqrt{2})}{16-18} = \frac{12-9\sqrt{2}-4\sqrt{2}+6}{-2} = \frac{1}{2}(13\sqrt{2}-18) \text{ or } \frac{13}{2}\sqrt{2}-9$$

$$e = \frac{1-\sqrt{2}}{3-2\sqrt{2}} \times \frac{3+2\sqrt{2}}{3+2\sqrt{2}} = \frac{(1-\sqrt{2})(3+2\sqrt{2})}{9-8} = 3 + 2\sqrt{2} - 3\sqrt{2} - 4 = -1 - \sqrt{2}$$

$$f = \frac{\sqrt{3}-5}{2\sqrt{3}-4} \times \frac{2\sqrt{3}+4}{2\sqrt{3}+4} = \frac{(\sqrt{3}-5)(2\sqrt{3}+4)}{12-16} = \frac{6+4\sqrt{3}-10\sqrt{3}-20}{-4} = \frac{1}{2}(7+3\sqrt{3})$$

$$g = \frac{2\sqrt{3}+3}{3-\sqrt{3}} \times \frac{3+\sqrt{3}}{3+\sqrt{3}} = \frac{(2\sqrt{3}+3)(3+\sqrt{3})}{9-3} = \frac{6\sqrt{3}+6+9+3\sqrt{3}}{6} = \frac{1}{2}(3\sqrt{3}+5)$$

$$h = \frac{3\sqrt{7}-2}{2\sqrt{7}-5} \times \frac{2\sqrt{7}+5}{2\sqrt{7}+5} = \frac{(3\sqrt{7}-2)(2\sqrt{7}+5)}{28-25} = \frac{42+15\sqrt{7}-4\sqrt{7}-10}{3} = \frac{1}{3}(32+11\sqrt{7})$$