**1** **a** $= \sqrt{9}\sqrt{3} + 2\sqrt{25}\sqrt{2}$
$= 10\sqrt{2} + 3\sqrt{3}$

**b** $= \sqrt{18} - \sqrt{48}$
$= \sqrt{9}\sqrt{2} - \sqrt{16}\sqrt{3}$
$= 3\sqrt{2} - 4\sqrt{3}$

**2** $x^2 - 2x = 12 - 2x$
$x^2 = 12$
$x = \pm\sqrt{12} = \pm 2\sqrt{3}$
$x > 0 \;\therefore\; x = 2\sqrt{3}$

**3** $25^x = (5^2)^x = 5^{4x+1}$
$5^{2x} = 5^{4x+1}$
$2x = 4x + 1$
$x = -\frac{1}{2}$

**4** **a** $= \sqrt[3]{8} \times \sqrt[3]{3} = 2\sqrt[3]{3}$

**b** $\sqrt[3]{81} = \sqrt[3]{27} \times \sqrt[3]{3} = 3\sqrt[3]{3}$
$\therefore\; \sqrt[3]{24} + \sqrt[3]{81} = 2\sqrt[3]{3} + 3\sqrt[3]{3} = 5\sqrt[3]{3}$
$= \sqrt[3]{125\times 3} = \sqrt[3]{375}$
$\therefore\; n = 375$

**5** $\frac{10\sqrt{3}}{\sqrt{15}} = \frac{10\sqrt{3}}{\sqrt{5}\sqrt{3}} = \frac{10}{\sqrt{5}} = \frac{10}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}} = 2\sqrt{5}$
$\frac{4}{\sqrt{5}-\sqrt{7}}\times\frac{\sqrt{5}+\sqrt{7}}{\sqrt{5}+\sqrt{7}} = \frac{4(\sqrt{5}+\sqrt{7})}{5-7} = -2\sqrt{5} - 2\sqrt{7}$
$\therefore\; \frac{10\sqrt{3}}{\sqrt{15}} + \frac{4}{\sqrt{5}-\sqrt{7}} = 2\sqrt{5} - 2\sqrt{5} - 2\sqrt{7}$
$= -2\sqrt{7}$ [ $k = -2$ ]

**6** **a** $= \sqrt{\frac{75}{2}} = \frac{5\sqrt{3}}{\sqrt{2}} = \frac{5\sqrt{3}}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}} = \frac{5}{2}\sqrt{6}$

**b** $= \sqrt{\frac{48}{5}} - \sqrt{\frac{20}{3}} = \frac{4\sqrt{3}}{\sqrt{5}} - \frac{2\sqrt{5}}{\sqrt{3}}$
$= \frac{4\sqrt{3}}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}} - \frac{2\sqrt{5}}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$
$= \frac{4}{5}\sqrt{15} - \frac{2}{3}\sqrt{15}$
$= \frac{2}{15}\sqrt{15}$

**7** **a** **i** $xy = 2^{t-1} \times 2^{3t} = 2^{4t-1}$
**ii** $2y^2 = 2 \times (2^{3t})^2 = 2 \times 2^{6t} = 2^{6t+1}$

**b** $2^{6t+1} - 2^{4t-1} = 0$
$2^{6t+1} = 2^{4t-1}$
$6t + 1 = 4t - 1$
$t = -1$

**8** $3x\sqrt{2} - \sqrt{2} = 4x + 6$
$x(3\sqrt{2} - 4) = 6 + \sqrt{2}$
$x = \frac{6+\sqrt{2}}{3\sqrt{2}-4} = \frac{6+\sqrt{2}}{3\sqrt{2}-4}\times\frac{3\sqrt{2}+4}{3\sqrt{2}+4} = \frac{(6+\sqrt{2})(3\sqrt{2}+4)}{18-16}$
$= \frac{1}{2}(18\sqrt{2} + 24 + 6 + 4\sqrt{2})$
$= \frac{1}{2}(30 + 22\sqrt{2})$
$= 15 + 11\sqrt{2}$

**9** **a** $6^{y+1} = 36^{x-2} = (6^2)^{x-2}$
$6^{y+1} = 6^{2x-4}$
$y + 1 = 2x - 4$
$y = 2x - 5$

**b** $x - \frac{1}{2}y = x - \frac{1}{2}(2x - 5) = x - x + \frac{5}{2} = \frac{5}{2}$
$\therefore\; 4^{x-\frac{1}{2}y} = 4^{\frac{5}{2}} = (\sqrt{4})^5 = 2^5 = 32$

**10** **a** $= 3 + 3\sqrt{2} - \sqrt{2} - 2$
$= 1 + 2\sqrt{2}$

**b** $= \frac{\sqrt{2}}{\sqrt{2}-1}\times\frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{\sqrt{2}(\sqrt{2}+1)}{2-1}$
$= \sqrt{2}(\sqrt{2} + 1)$
$= 2 + \sqrt{2}$

**11** $(2^4)^{x+1} = (2^3)^{2x+1}$
$2^{4x+4} = 2^{6x+3}$
$4x + 4 = 6x + 3$
$x = \frac{1}{2}$

**12** $a^2 - 4a\sqrt{3} + 12 = b - 20\sqrt{3}$
$a$ and $b$ integers $\therefore$ $-4a = -20$
$a = 5$
also $a^2 + 12 = b$
$b = 37$

**13** **a** $(2^{-2})^{t-3} = 2^3$
$2^{6-2t} = 2^3$
$6 - 2t = 3$
$t = \frac{3}{2}$

**b** $(3^{-1})^y = (3^3)^{y+1}$
$3^{-y} = 3^{3y+3}$
$-y = 3y + 3$
$y = -\frac{3}{4}$

**14** **a** $= 2\sqrt{5}(\sqrt{5} - 3)$
$= 10 - 6\sqrt{5}$

**b** $= 3 + 2\sqrt{5} - 3\sqrt{5} - 10$
$= -7 - \sqrt{5}$

**c** $= \frac{1+\sqrt{5}}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2} = \frac{(1+\sqrt{5})(\sqrt{5}+2)}{5-4}$
$= (1 + \sqrt{5})(\sqrt{5} + 2)$
$= \sqrt{5} + 2 + 5 + 2\sqrt{5}$
$= 7 + 3\sqrt{5}$

**15** **a** $a = (b^{\frac{3}{4}})^3 = b^{\frac{9}{4}}$
$a^{\frac{1}{2}} = (b^{\frac{9}{4}})^{\frac{1}{2}} = b^{\frac{9}{8}}$

**b** $b = (a^{\frac{1}{3}})^{\frac{4}{3}} = a^{\frac{4}{9}}$
$b^{\frac{1}{2}} = (a^{\frac{4}{9}})^{\frac{1}{2}} = a^{\frac{2}{9}}$

**16** **a** area $= \frac{1}{2}(2\sqrt{3} - 1)(\sqrt{3} + 2)$
$= \frac{1}{2}(6 + 4\sqrt{3} - \sqrt{3} - 2)$
$= \frac{1}{2}(4 + 3\sqrt{3})$ or $2 + \frac{3}{2}\sqrt{3}$

**b** $AC^2 = (2\sqrt{3} - 1)^2 + (\sqrt{3} + 2)^2$
$= 12 - 4\sqrt{3} + 1 + 3 + 4\sqrt{3} + 4 = 20$
$\therefore$ $AC = \sqrt{20} = \sqrt{4}\sqrt{5} = 2\sqrt{5}$

**c** $\tan(\angle ACB) = \frac{2\sqrt{3}-1}{\sqrt{3}+2} \times \frac{\sqrt{3}-2}{\sqrt{3}-2} = \frac{(2\sqrt{3}-1)(\sqrt{3}-2)}{3-4}$
$= -(2\sqrt{3} - 1)(\sqrt{3} - 2)$
$= -(6 - 4\sqrt{3} - \sqrt{3} + 2)$
$= -(8 - 5\sqrt{3}) = 5\sqrt{3} - 8$

**17** **a** **i** $2^{x+2} = 2^2 \times 2^x = 4y$
**ii** $4^x = (2^2)^x = 2^{2x} = (2^x)^2 = y^2$

**b** $y^2 - 4y = 0$
$y(y - 4) = 0$
$y = 0$ or $4$
$2^x = 0$ (no solutions) or $2^x = 4$
$x = 2$

**18** $5\sqrt{3} = 2(1 + \sqrt{3})^2 + p(1 + \sqrt{3}) + q$
$5\sqrt{3} = 2 + 4\sqrt{3} + 6 + p + p\sqrt{3} + q$
$p$, $q$ rational $\therefore$ $5\sqrt{3} = 4\sqrt{3} + p\sqrt{3}$
$p = 1$
and $0 = 2 + 6 + p + q$
$q = -9$