1 Express in the form $(x + a)^2 + b$

a $x^2 + 2x + 4$ **b** $x^2 - 2x + 4$ **c** $x^2 - 4x + 1$ **d** $x^2 + 6x$

e $x^2 + 4x + 8$ **f** $x^2 - 8x - 5$ **g** $x^2 + 12x + 30$ **h** $x^2 - 10x + 25$

i $x^2 + 6x - 9$ **j** $18 - 4x + x^2$ **k** $x^2 + 3x + 3$ **l** $x^2 + x - 1$

m $x^2 - 18x + 100$ **n** $x^2 - x - \frac{1}{2}$ **o** $20 + 9x + x^2$ **p** $x^2 - 7x - 2$

q $5 - 3x + x^2$ **r** $x^2 - 11x + 37$ **s** $x^2 + \frac{2}{3}x + 1$ **t** $x^2 - \frac{1}{2}x - \frac{1}{4}$

2 Express in the form $a(x + b)^2 + c$

a $2x^2 + 4x + 3$ **b** $2x^2 - 8x - 7$ **c** $3 - 6x + 3x^2$ **d** $4x^2 + 24x + 11$

e $-x^2 - 2x - 5$ **f** $1 + 10x - x^2$ **g** $2x^2 + 2x - 1$ **h** $3x^2 - 9x + 5$

i $3x^2 - 24x + 48$ **j** $3x^2 - 15x$ **k** $70 + 40x + 5x^2$ **l** $2x^2 + 5x + 2$

m $4x^2 + 6x - 7$ **n** $-2x^2 + 4x - 1$ **o** $4 - 2x - 3x^2$ **p** $\frac{1}{3}x^2 + \frac{1}{2}x - \frac{1}{4}$

3 Solve each equation by completing the square, giving your answers as simply as possible in terms of surds where appropriate.

a $y^2 - 4y + 2 = 0$ **b** $p^2 + 2p - 2 = 0$ **c** $x^2 - 6x + 4 = 0$ **d** $7 + 10r + r^2 = 0$

e $x^2 - 2x = 11$ **f** $a^2 - 12a - 18 = 0$ **g** $m^2 - 3m + 1 = 0$ **h** $9 - 7t + t^2 = 0$

i $u^2 + 7u = 44$ **j** $2y^2 - 4y + 1 = 0$ **k** $3p^2 + 18p = -23$ **l** $2x^2 + 12x = 9$

m $-m^2 + m + 1 = 0$ **n** $4x^2 + 49 = 28x$ **o** $1 - t - 3t^2 = 0$ **p** $2a^2 - 7a + 4 = 0$

4 By completing the square, find the maximum or minimum value of y and the value of x for which this occurs. State whether your value of y is a maximum or a minimum in each case.

a $y = x^2 - 2x + 7$ **b** $y = x^2 + 2x - 3$ **c** $y = 1 - 6x + x^2$

d $y = x^2 + 10x + 35$ **e** $y = -x^2 + 4x + 4$ **f** $y = x^2 + 3x - 2$

g $y = 2x^2 + 8x + 5$ **h** $y = -3x^2 + 6x$ **i** $y = 7 - 5x - x^2$

j $y = 4x^2 - 12x + 9$ **k** $y = 4x^2 + 20x - 8$ **l** $y = 17 - 2x - 2x^2$

5 Sketch each curve showing the exact coordinates of its turning point and the point where it crosses the y-axis.

a $y = x^2 - 4x + 3$ **b** $y = x^2 + 2x - 24$ **c** $y = x^2 - 2x + 5$

d $y = 30 + 8x + x^2$ **e** $y = x^2 + 2x + 1$ **f** $y = 8 + 2x - x^2$

g $y = -x^2 + 8x - 7$ **h** $y = -x^2 - 4x - 7$ **i** $y = x^2 - 5x + 4$

j $y = x^2 + 3x + 3$ **k** $y = 3 + 8x + 4x^2$ **l** $y = -2x^2 + 8x - 15$

m $y = 1 - x - 2x^2$ **n** $y = 25 - 20x + 4x^2$ **o** $y = 3x^2 - 4x + 2$

6 **a** Express $x^2 - 4\sqrt{2}\,x + 5$ in the form $a(x + b)^2 + c$.

b Write down an equation of the line of symmetry of the curve $y = x^2 + 4\sqrt{2}\,x + 5$.

7 $\mathrm{f}(x) \equiv x^2 + 2kx - 3$.

By completing the square, find the roots of the equation $\mathrm{f}(x) = 0$ in terms of the constant k.

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