1 **a** $= 2x(10 - x - 3x^2)$
$= 2x(2 + x)(5 - 3x)$

b $2x(2 + x)(5 - 3x) = 0$
$x = -2, 0$ or $\frac{5}{3}$

2 **a** $AB^2 = (6 + 2)^2 + (k - 1)^2 = 64 + k^2 - 2k + 1$
$= k^2 - 2k + 65$

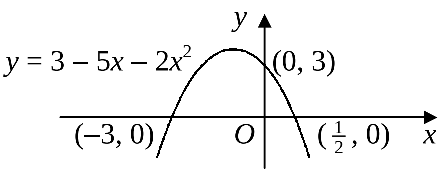
b $k^2 - 2k + 65 = 10^2 = 100$
$k^2 - 2k - 35 = 0$
$(k + 5)(k - 7) = 0$
$k = -5$ or 7

3 **a** $x^2 - 5 = 4x$
$x^2 - 4x - 5 = 0$
$(x + 1)(x - 5) = 0$
$x = -1$ or 5

b $9 - (5 - x) = 2x(5 - x)$
$2x^2 - 9x + 4 = 0$
$(2x - 1)(x - 4) = 0$
$x = \frac{1}{2}$ or 4

4 **a** $y = -2[x^2 + \frac{5}{2}x] + 3$
$= -2[(x + \frac{5}{4})^2 - \frac{25}{16}] + 3$
$= -2(x + \frac{5}{4})^2 + \frac{49}{8}$
$\therefore$ turning point is $(-\frac{5}{4}, \frac{49}{8})$

b $3 - 5x - 2x^2 = 0$
$2x^2 + 5x - 3 = 0$
$(2x - 1)(x + 3) = 0$, $x = -3$ or $\frac{1}{2}$



5 $x = \dfrac{-5\sqrt{2} \pm \sqrt{50 + 48}}{4}$
$= \dfrac{-5\sqrt{2} \pm \sqrt{98}}{4}$
$= \dfrac{-5\sqrt{2} \pm 7\sqrt{2}}{4}$
$= -3\sqrt{2}$ or $\frac{1}{2}\sqrt{2}$

6 **a** $y = 3[x^2 - 3x] + k = 3[(x - \frac{3}{2})^2 - \frac{9}{4}] + k$
$= 3(x - \frac{3}{2})^2 - \frac{27}{4} + k$
$\therefore$ x-coordinate of $P = \frac{3}{2}$

b y-coord of $P = k - \frac{27}{4} = \frac{17}{4}$ $\therefore$ $k = 11$
$\therefore$ curve is $y = 3x^2 - 9x + 11$
$\therefore$ coordinates of Q are $(0, 11)$

7 $y^2 - 10y + 16 = 0$
$(y - 2)(y - 8) = 0$
$y = 2^x = 2$ or 8
$x = 1$ or 3

8 equal roots $\therefore$ $b^2 - 4ac = 0$
$4 - 4k(3 - 2k) = 0$
$2k^2 - 3k + 1 = 0$
$(2k - 1)(k - 1) = 0$
$k = \frac{1}{2}$ or 1

9 **a** $f(x) = -[x^2 - 4x] + 3$
$= -[(x - 2)^2 - 4] + 3$
$= -(x - 2)^2 + 7$

b turning point is $(2, 7)$

c $-(x - 2)^2 + 7 = 2$
$(x - 2)^2 = 5$
$x = 2 \pm \sqrt{5}$

10 **a** $x = \dfrac{5 \pm \sqrt{25 - 12}}{6}$
$= \frac{1}{6}(5 \pm \sqrt{13})$

b $x(x - 1) = 3(x + 2)$
$x^2 - 4x - 6 = 0$
$x = \dfrac{4 \pm \sqrt{16 + 24}}{2} = \dfrac{4 \pm 2\sqrt{10}}{2}$
$= 2 \pm \sqrt{10}$

11 **a** $(x - 2k)^2 - 4k^2 + 6 = 0$

$(x - 2k)^2 = 4k^2 - 6$

$x - 2k = \pm\sqrt{4k^2 - 6}$

$x = 2k \pm \sqrt{4k^2 - 6}$

b $k = 3$

$\therefore\ x = 6 \pm \sqrt{36 - 6}$

$= 6 \pm \sqrt{30}$

12 **a** $x^2 - 6x - 3 = 0$

$x = \frac{6 \pm \sqrt{36+12}}{2} = \frac{6 \pm 4\sqrt{3}}{2}$

$= 3 \pm 2\sqrt{3}$

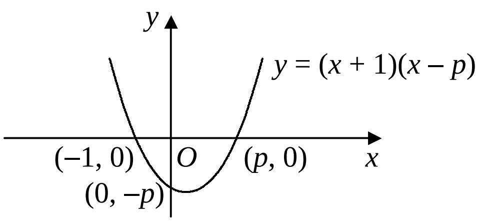
b $y(2y^2 + y - 15) = 0$

$y(2y - 5)(y + 3) = 0$

$y = -3, 0$ or $\frac{5}{2}$

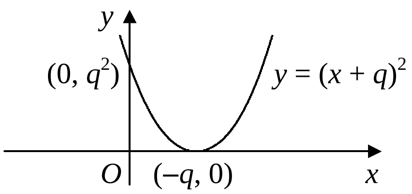
13 **a** $x = 0 \ \Rightarrow\ y = -p$

$y = 0 \ \Rightarrow\ x = -1$ or p



b $x = 0 \ \Rightarrow\ y = q^2$

$y = 0 \ \Rightarrow\ x = -q \quad [-q > 0]$



14 **a** $f(x) = 2[x^2 - 3x] + 5$

$= 2[(x - \frac{3}{2})^2 - \frac{9}{4}] + 5$

$= 2(x - \frac{3}{2})^2 + \frac{1}{2}$

$\therefore\ A = 2,\ B = -\frac{3}{2},\ C = \frac{1}{2}$

b minimum value of $f(x) = \frac{1}{2}$

15 **a** $x^{\frac{2}{3}} = (x^{\frac{1}{3}})^2 = t^2$

b let $t = x^{\frac{1}{3}} \ \Rightarrow\ 2t^2 + t - 6 = 0$

$(2t - 3)(t + 2) = 0$

$t = -2$ or $\frac{3}{2}$

but $x = t^3 \quad \therefore \quad x = -8$ or $\frac{27}{8}$

16 **a** $= (k - 4)^2 - 16 + 20$

$= (k - 4)^2 + 4$

b $x^2 - kx + 2k - 5 = 0$

discriminant $= b^2 - 4ac$

$= k^2 - 4(2k - 5)$

$= k^2 - 8k + 20$

using **a** $= (k - 4)^2 + 4$

for all real k, $(k - 4)^2 \geq 0$

$\therefore$ discriminant > 0

$\therefore$ real and distinct roots for all real k

17 **a** $(x^2 + 2x - 3)(x^2 - 3x - 4) \equiv x^2(x^2 - 3x - 4) + 2x(x^2 - 3x - 4) - 3(x^2 - 3x - 4)$

$\equiv x^4 - 3x^3 - 4x^2 + 2x^3 - 6x^2 - 8x - 3x^2 + 9x + 12$

$\equiv x^4 - x^3 - 13x^2 + x + 12$

b $(x^2 + 2x - 3)(x^2 - 3x - 4) = 0$

$(x + 3)(x - 1)(x + 1)(x - 4) = 0$

$x = -3, -1, 1$ or 4