

1 a $= (\frac{3}{2})^2 = \frac{9}{4}$ or $2\frac{1}{4}$

b $x^{\frac{3}{2}} = 27$
 $x^{\frac{1}{2}} = \sqrt[3]{27} = 3$
 $x = 3^2 = 9$

3 a $= 8\sqrt{3} - 4\sqrt{3} + 5\sqrt{3}$
 $= 9\sqrt{3}$

b $= 10 - 4\sqrt{3} + 5\sqrt{3} - 6$
 $= 4 + \sqrt{3}$

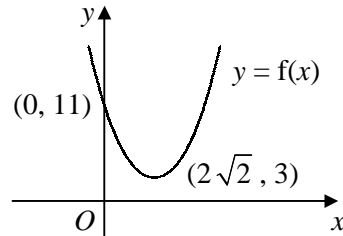
2 $x = 16 - 3y$

sub. $(16 - 3y)^2 - y(16 - 3y) + 2y^2 = 46$
 $y^2 - 8y + 15 = 0$
 $(y - 3)(y - 5) = 0$
 $y = 3$ or 5
 $\therefore x = 1, y = 5$ or $x = 7, y = 3$

4 a $f(x) = (x - 2\sqrt{2})^2 - 8 + 11$
 $= (x - 2\sqrt{2})^2 + 3$

$\therefore a = 1, b = -2\sqrt{2}$ and $c = 3$

b turning point is $(2\sqrt{2}, 3)$

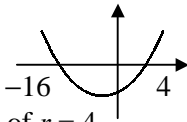


5 a $S.A = 2\pi r^2 + 2\pi rh = 2\pi r^2 + 24\pi r$
 $S.A \leq 128\pi \therefore 2\pi r^2 + 24\pi r \leq 128\pi$
 $r^2 + 12r \leq 64$
 $r^2 + 12r - 64 \leq 0$

b $(r + 16)(r - 4) \leq 0$

$-16 \leq r \leq 4$

\therefore maximum value of $r = 4$



6 $8x\sqrt{x} = 4x$

$4x(2\sqrt{x} - 1) = 0$

$x \neq 0 \therefore \sqrt{x} = \frac{1}{2}$

$x = (\frac{1}{2})^2 = \frac{1}{4}$

7 a $x = 5$

b $(2^5)^{y+1} = (2^2)^y$

$5y + 5 = 2y$

$y = -\frac{5}{3}$

8 a $t^2 - 5t$

b $t^2 - 5t + 6 = 0$

$(t - 2)(t - 3) = 0$

$t = 2, 3$

$x = t^2 = 4, 9$

9 $x^2 + kx + 3 + k^2 = 0$

$\Rightarrow (x + \frac{1}{2}k)^2 - \frac{1}{4}k^2 + 3 + k^2 = 0$

$\Rightarrow x + \frac{1}{2}k = \pm \sqrt{-\frac{3}{4}k^2 - 3}$

$\Rightarrow x = -\frac{1}{2}k \pm \sqrt{-\frac{3}{4}k^2 - 3}$

real $k \Rightarrow k^2 \geq 0$

$\Rightarrow -\frac{3}{4}k^2 - 3 < 0$

\therefore no real roots

10 a $(2^3)^{2x-1} = 2^5$

$6x - 3 = 5$

$x = \frac{4}{3}$

b $(3^{-1})^{y-2} = 3^4$

$-y + 2 = 4$

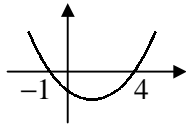
$y = -2$

11 $2x^2 - 7x < x^2 - 4x + 4$

$$x^2 - 3x - 4 < 0$$

$$(x + 1)(x - 4) < 0$$

$$-1 < x < 4$$



12 $\frac{2}{3\sqrt{2}-4} = \frac{2}{3\sqrt{2}-4} \times \frac{3\sqrt{2}+4}{3\sqrt{2}+4} = \frac{2(3\sqrt{2}+4)}{18-16} = 3\sqrt{2} + 4$

$$\frac{3-\sqrt{2}}{\sqrt{2}+1} = \frac{3-\sqrt{2}}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{(3-\sqrt{2})(\sqrt{2}-1)}{2-1}$$

$$= (3 - \sqrt{2})(\sqrt{2} - 1)$$

$$= 3\sqrt{2} - 3 - 2 + \sqrt{2} = 4\sqrt{2} - 5$$

$$\therefore \frac{2}{3\sqrt{2}-4} - \frac{3-\sqrt{2}}{\sqrt{2}+1} = 3\sqrt{2} + 4 - (4\sqrt{2} - 5)$$

$$= 9 - \sqrt{2}$$

13 a $(3y - 1)(2y + 9) = 0$

$$y = -\frac{9}{2} \text{ or } \frac{1}{3}$$

b equal roots

$$\therefore b^2 - 4ac = 0$$

$$k^2 - 64 = 0$$

$$k = \pm 8$$

14 a i $4^x = (2^2)^x = 2^{2x} = (2^x)^2 = y^2$

ii $2^{x-1} = 2^{-1} \times 2^x = \frac{1}{2}y$

b let $y = 2^x \Rightarrow y^2 - 9(\frac{1}{2}y) + 2 = 0$

$$2y^2 - 9y + 4 = 0$$

$$(2y - 1)(y - 4) = 0$$

$$y = 2^x = \frac{1}{2} \text{ or } 4$$

$$x = -1 \text{ or } 2$$

15 $x = 3y + 1$

sub.

$$(3y + 1)^2 + 2y(3y + 1) + y^2 = 9$$

$$2y^2 + y - 1 = 0$$

$$(2y - 1)(y + 1) = 0$$

$$y = -1 \text{ or } \frac{1}{2}$$

$$\therefore (-2, -1) \text{ and } (\frac{5}{2}, \frac{1}{2})$$

16 a $(x + \frac{1}{2}a)^2 - \frac{1}{4}a^2 + b = 0$

$$(x + \frac{1}{2}a)^2 = \frac{1}{4}a^2 - b = \frac{a^2 - 4b}{4}$$

$$x + \frac{1}{2}a = \pm \sqrt{\frac{a^2 - 4b}{4}} = \pm \frac{\sqrt{a^2 - 4b}}{2}$$

$$x = -\frac{1}{2}a \pm \frac{\sqrt{a^2 - 4b}}{2}$$

$$x = \frac{-a \pm \sqrt{a^2 - 4b}}{2}$$

b for repeated root, $a^2 - 4b = 0$

$$\Rightarrow b = \frac{1}{4}a^2$$

17 a $f(x) = 2[x^2 - 6x] + 19$

$$= 2[(x - 3)^2 - 9] + 19$$

$$= 2(x - 3)^2 + 1$$

$$\text{real } x \Rightarrow (x - 3)^2 \geq 0$$

$$\Rightarrow 2(x - 3)^2 + 1 \geq 1$$

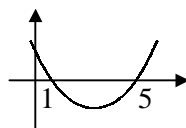
$$\Rightarrow f(x) \geq 1$$

b $2x^2 - 12x + 19 < 9$

$$x^2 - 6x + 5 < 0$$

$$(x - 1)(x - 5) < 0$$

$$1 < x < 5$$



18 a $= 1 - 2\sqrt{5} + 5$

$$= 6 - 2\sqrt{5}$$

b $y^2 = \frac{1}{2}(6 - 2\sqrt{5}) = \frac{1}{2}(1 - \sqrt{5})^2$

$$y = \pm \frac{1}{\sqrt{2}}(1 - \sqrt{5})$$

$$y = \pm \frac{1}{2}\sqrt{2}(1 - \sqrt{5})$$

$$y = \pm \frac{1}{2}(\sqrt{2} - \sqrt{10})$$

$$y = \frac{1}{2}\sqrt{2} - \frac{1}{2}\sqrt{10} \text{ or } -\frac{1}{2}\sqrt{2} + \frac{1}{2}\sqrt{10}$$