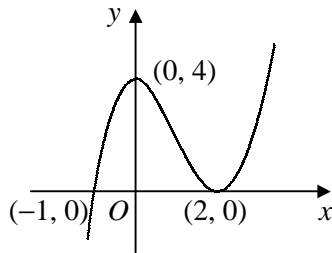


1 a



$$\begin{aligned} \text{b } f(x) &= (x+1)(x^2 - 4x + 4) \\ &= x^3 - 4x^2 + 4x + x^2 - 4x + 4 \\ &= x^3 - 3x^2 + 4 \end{aligned}$$

$$f'(x) = 3x^2 - 6x$$

$$\text{c } x = 1 \quad \therefore y = 2 \times (-1)^2 = 2$$

$$\text{grad} = 3 - 6 = -3$$

$$\begin{aligned} \therefore y - 2 &= -3(x - 1) \\ y - 2 &= -3x + 3 \\ y &= 5 - 3x \end{aligned}$$

3

$$\text{a } x^2 + x - 2 = 0$$

$$(x+2)(x-1) = 0$$

$$x = -2, 1 \quad a < b \quad \therefore a = -2, b = 1$$

$$\text{b } \frac{dy}{dx} = 2x + 1$$

$$\text{grad at A} = -3$$

$$\therefore \text{grad of normal} = \frac{1}{3}$$

$$\therefore y - 0 = \frac{1}{3}(x + 2)$$

$$3y = x + 2$$

$$x - 3y + 2 = 0$$

$$\text{c } \text{grad at B} = 3$$

$$\text{tangent at B: } y - 0 = 3(x - 1)$$

$$y = 3x - 3$$

$$\text{at C, } x - 3(3x - 3) + 2 = 0$$

$$x = \frac{11}{8}$$

$$\therefore C \left( \frac{11}{8}, \frac{9}{8} \right)$$

5

$$\text{a } \frac{dy}{dx} = -24x^{-3}$$

$$\text{at A, } y = 3, \text{ grad} = -3$$

$$\therefore y - 3 = -3(x - 2)$$

$$3x + y - 9 = 0$$

$$\text{b } \text{tangent:}$$

$$x = -1 \Rightarrow -3 + y - 9 = 0 \Rightarrow y = 12$$

curve:

$$x = -1 \Rightarrow y = \frac{12}{1} \Rightarrow y = 12$$

$$\therefore \text{tangent intersects curve at } (-1, 12)$$

2

$$\text{a } \frac{dy}{dx} = 1 - \frac{3}{2}x^{-\frac{1}{2}}$$

$$\text{grad at P} = \frac{1}{4}$$

$$\therefore y - 1 = \frac{1}{4}(x - 4)$$

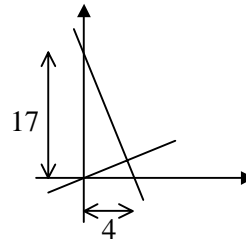
$$y = \frac{1}{4}x \text{ which passes through } (0, 0)$$

$$\text{b } \text{grad of normal} = -4$$

$$\therefore y - 1 = -4(x - 4) \quad [y = 17 - 4x]$$

$$\text{at Q, } x = 0 \Rightarrow y = 17$$

$$\therefore \text{area} = \frac{1}{2} \times 17 \times 4 = 34$$



4

$$y = \frac{1}{3}x^{\frac{3}{2}} - 2x^{\frac{1}{2}} - x^{-\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{1}{2}x^{\frac{1}{2}} - x^{-\frac{1}{2}} + \frac{1}{2}x^{-\frac{3}{2}}$$

$$= \frac{x^2 - 2x + 1}{2x^{\frac{3}{2}}}$$

$$= \frac{(x-1)^2}{2x^{\frac{3}{2}}} \quad [a = -1, b = 2]$$

6

$$\text{a } \frac{dy}{dx} = 3 + 2kx - 3x^2$$

$$\text{at P, } 3 - 2k - 3 = -6$$

$$k = 3$$

$$\text{b } y = 2 + 3x + 3x^2 - x^3 \quad \therefore P(-1, 3)$$

$$\text{at Q, } 3 + 6x - 3x^2 = -6$$

$$x^2 - 2x - 3 = 0$$

$$(x+1)(x-3) = 0$$

$$x = -1 \text{ (at P) or } 3 \quad \therefore Q(3, 11)$$

$$PQ = \sqrt{16 + 64} = \sqrt{80} = 4\sqrt{5}$$

$$7 \quad = \frac{d}{dx}(x^2 + \frac{1}{2}x^{-1})$$

$$= 2x - \frac{1}{2}x^{-2}$$

$$8 \quad \text{a} \quad \frac{dy}{dx} = 4x - 7$$

$$\text{at } A, y = -5, \text{ grad} = 1$$

$$\therefore y + 5 = 1(x - 2)$$

$$[y = x - 7]$$

$$\text{b} \quad \text{grad of normal at } B = 1$$

$$\therefore \text{grad of curve at } B = -1$$

$$\therefore 4x - 7 = -1$$

$$x = \frac{3}{2}, y = 2(\frac{3}{2}) - 7(\frac{3}{2}) + 1 = -5$$

$$\therefore B(\frac{3}{2}, -5)$$

$$9 \quad \text{a} \quad \frac{dy}{dx} = 2x + \frac{3}{2}x^{-\frac{1}{2}}$$

$$\text{b} \quad \frac{d^2y}{dx^2} = 2 - \frac{3}{4}x^{-\frac{3}{2}}$$

$$\therefore 2x \frac{d^2y}{dx^2} + \frac{dy}{dx} - 6x$$

$$= 2x(2 - \frac{3}{4}x^{-\frac{3}{2}}) + 2x + \frac{3}{2}x^{-\frac{1}{2}} - 6x$$

$$= 4x - \frac{3}{2}x^{-\frac{1}{2}} + 2x + \frac{3}{2}x^{-\frac{1}{2}} - 6x$$

$$= 0$$

$$10 \quad \text{a} \quad \frac{dy}{dx} = -4x^{-2}$$

$$\text{grad at } M = -\frac{1}{4}$$

$$\therefore \text{grad of normal} = 4$$

$$\therefore y - 3 = 4(x - 4) \quad [y = 4x - 13]$$

$$\text{b} \quad 4x - 13 = 2 + \frac{4}{x}$$

$$4x^2 - 15x - 4 = 0$$

$$(4x + 1)(x - 4) = 0$$

$$x = 4 \text{ (at } M) \text{ or } -\frac{1}{4}$$

$$\therefore N(-\frac{1}{4}, -14)$$

$$11 \quad \text{a} \quad \frac{dy}{dx} = 3x^2 - 6x - 8$$

$$\text{grad at } P = 1$$

$$\therefore y - 8 = 1(x + 1) \quad [y = x + 9]$$

$$\text{b} \quad \text{at } Q, \quad 3x^2 - 6x - 8 = 1$$

$$x^2 - 2x - 3 = 0$$

$$(x + 1)(x - 3) = 0$$

$$x = -1 \text{ at } P \therefore Q(3, -20)$$

$$\therefore y + 20 = 1(x - 3) \quad [y = x - 23]$$

$$\text{c} \quad \text{grad normal} = -1$$

$$\therefore y - 8 = -(x + 1) \quad [y = 7 - x]$$

$$\text{d} \quad \text{normal at } P \text{ meets } m \text{ when}$$

$$7 - x = x - 23$$

$$x = 15 \therefore (15, -8)$$

$$\text{dist between lines} = \text{dist } P \text{ to } (15, -8)$$

$$= \sqrt{16^2 + 16^2} = \sqrt{16^2 \times 2} = 16\sqrt{2}$$

$$12 \quad \text{a} \quad y = kx^{\frac{1}{2}} - x^{\frac{3}{2}}$$

$$\frac{dy}{dx} = \frac{1}{2}kx^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}}$$

$$\text{at } P, \quad \frac{1}{2}k(\frac{1}{\sqrt{2}}) - \frac{3}{2}(\sqrt{2}) = \sqrt{2}$$

$$k - 6 = 4$$

$$k = 10$$

$$\text{b} \quad y = \sqrt{x}(10 - x)$$

$$\text{at } P, y = \sqrt{2}(10 - 2) = 8\sqrt{2}$$

$$\text{grad of normal} = -\frac{1}{\sqrt{2}}$$

$$\therefore y - 8\sqrt{2} = -\frac{1}{\sqrt{2}}(x - 2)$$

$$\sqrt{2}y - 16 = -x + 2$$

$$x + \sqrt{2}y = 18 \quad [c = 18]$$