C1 GRAPHS OF FUNCTIONS *Worksheet A*

1 Sketch and label each pair of graphs on the same set of axes showing the coordinates of any points where the graphs intersect. Write down the equations of any asymptotes.

a $y = x^2$ and $y = x^3$

b $y = x^2$ and $y = x^4$

c $y = \frac{1}{x}$ and $y = \frac{1}{x^2}$

d $y = x$ and $y = \sqrt{x}$

e $y = x^2$ and $y = 3x^2$

f $y = \frac{1}{x}$ and $y = \frac{2}{x}$

2 $\mathrm{f}(x) = (x - 1)(x - 3)(x - 4)$.

a Find f(0).

b Write down the solutions of the equation $\mathrm{f}(x) = 0$.

c Sketch the curve $y = \mathrm{f}(x)$.

3 Sketch each graph showing the coordinates of any points of intersection with the coordinate axes.

a $y = (x + 1)(x - 1)(x - 3)$

b $y = 2x(x - 1)(x - 5)$

c $y = -(x + 2)(x + 1)(x - 2)$

d $y = x^2(x - 4)$

e $y = 3x(2 + x)(1 - x)$

f $y = (x + 2)(x - 1)^2$

4 **a** Factorise fully $x^3 + 6x^2 + 9x$.

b Hence, sketch the curve $y = x^3 + 6x^2 + 9x$, showing the coordinates of any points where the curve meets the coordinate axes.

5 Given that the constants p and q are such that $p > q > 0$, sketch each of the following graphs showing the coordinates of any points of intersection with the coordinate axes.

a $y = (x - p)(x - q)^2$

b $y = (x - p)(x^2 - q^2)$

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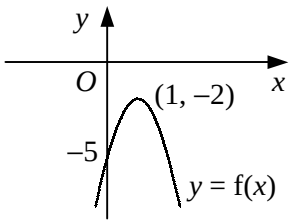

The diagram shows the curve with equation $y = \mathrm{f}(x)$ which has a turning point at (1, −2) and crosses the y-axis at the point (0, −5).

Given that $\mathrm{f}(x)$ is a quadratic function, find an expression for $\mathrm{f}(x)$.

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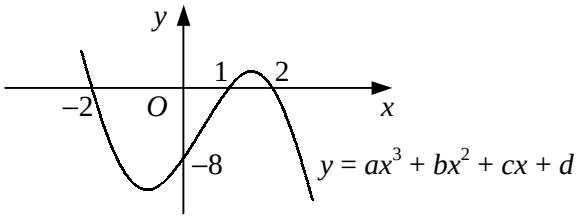

The diagram shows the curve with equation $y = ax^3 + bx^2 + cx + d$.

Given that the curve crosses the y-axis at the point (0, −8) and crosses the x-axis at the points (−2, 0), (1, 0) and (2, 0), find the values of the constants a, b, c and d.

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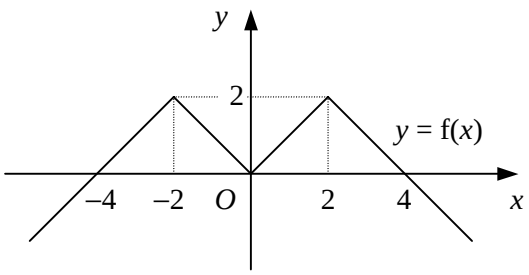

The diagram shows the graph of $y = f(x)$.

Use the graph to write down the number of solutions that exist to each of the following equations.

a $f(x) = 1$ **b** $f(x) = 3$ **c** $f(x) = -1$ **d** $f(x) = 0$

9 **a** Sketch on the same set of axes the graphs of $y = x^2$ and $y = 1 - 2x$.

b Hence state the number of roots that the equation $x^2 + 2x - 1 = 0$ has and give a reason for your answer.

10 **a** Find the coordinates of the turning point of the curve $y = x^2 + 2x - 3$.

b By sketching two suitable graphs on the same set of axes, show that the equation

$$x^2 + 2x - 3 - \frac{1}{x} = 0$$

has one positive and two negative real roots.

11 Show that the line $y = x - 3$ is a tangent to the curve $y = x^2 - 5x + 6$.

12 **a** Solve the simultaneous equations

$$y = 3x + 7$$
$$y = x^2 + 5x + 8$$

b Hence, describe the geometrical relationship between the straight line $y = 3x + 7$ and the curve $y = x^2 + 5x + 8$.

13 **a** Find the coordinates of the points where the straight line $y = x + 6$ meets the curve $y = x^3 - 4x^2 + x + 6$.

b Given that

$$x^3 - 4x^2 + x + 6 \equiv (x + 1)(x - 2)(x - 3),$$

sketch the straight line $y = x + 6$ and the curve $y = x^3 - 4x^2 + x + 6$ on the same diagram, showing the coordinates of the points where the curve crosses the coordinate axes.

14 Find the value of the constant k such that the straight line with equation $y = 3x + k$ is a tangent to the curve with equation $y = 2x^2 - 5x + 1$.

15 Find the set of values of the constant a for which the line $y = 2 - 5x$ intersects the curve $y = x^2 + ax + 18$ at two points.

16 The curve C has the equation $y = x^2 - 2x + 6$.

a Find the values of p for which the line $y = px + p$ is a tangent to the curve C.

b Prove that there are no real values of q for which the line $y = qx + 7$ is a tangent to the curve C.