

$$1 \quad = \frac{1}{3}x^3 + 4x^{\frac{3}{2}} - 3x + c$$

$$2 \quad \mathbf{a} \quad f(x) = \int (1 - 6x^{-3}) \, dx$$

$$= x + 3x^{-2} + c$$

$$(1, -2) \Rightarrow -2 = 1 + 3 + c$$

$$c = -6$$

$$\therefore f(x) = x - 6 + \frac{3}{x^2}$$

$$\mathbf{b} \quad x = 2 \Rightarrow y = -\frac{13}{4}, \text{ grad} = \frac{1}{4}$$

$$\therefore \text{grad of normal} = -4$$

$$\therefore y + \frac{13}{4} = -4(x - 2)$$

$$4y + 13 = -16x + 32$$

$$16x + 4y - 19 = 0$$

$$3 \quad \mathbf{a} \quad f(x) = \int (3x^2 + 2x - 5) \, dx$$

$$= x^3 + x^2 - 5x + c$$

$$(3, 22) \Rightarrow 22 = 27 + 9 - 15 + c$$

$$c = 1$$

$$\therefore f(x) = x^3 + x^2 - 5x + 1$$

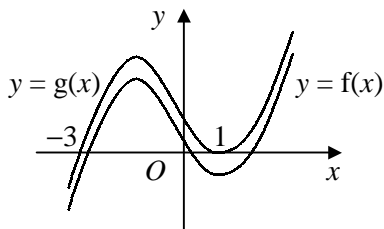
$$\mathbf{b} \quad g(x) = (x + 3)(x^2 - 2x + 1)$$

$$= x^3 - 2x^2 + x + 3x^2 - 6x + 3$$

$$= x^3 + x^2 - 5x + 3$$

$$= f(x) + 2$$

c



$$4 \quad \mathbf{a} \quad = 2x + 6x^{-3}$$

$$\mathbf{b} \quad = \frac{1}{3}x^3 + 3x^{-1} + c$$

$$5 \quad \text{grad of tangent} = 12 - 8 - 1 = 3$$

tangent passes through (0, 0)

$$\therefore \text{tangent: } y = 3x$$

$$\text{when } x = 2, y = 6$$

$$\therefore \text{curve passes through } (2, 6)$$

$$\text{curve: } y = \int (3x^2 - 4x - 1) \, dx$$

$$y = x^3 - 2x^2 - x + c$$

$$(2, 6) \Rightarrow 6 = 8 - 8 - 2 + c$$

$$c = 8$$

$$\therefore y = x^3 - 2x^2 - x + 8$$

$$6 \quad \mathbf{a} \quad = 3\sqrt{2} - \frac{2}{\sqrt{2}}$$

$$= 3\sqrt{2} - \sqrt{2}$$

$$= 2\sqrt{2}$$

$$\mathbf{b} \quad y = \int (3x^{\frac{1}{2}} - 2x^{-\frac{1}{2}}) \, dx$$

$$= 2x^{\frac{3}{2}} - 4x^{\frac{1}{2}} + c$$

$$(4, 7) \Rightarrow 7 = 2(8) - 4(2) + c$$

$$7 = 16 - 8 + c$$

$$c = -1$$

$$\therefore y = 2x^{\frac{3}{2}} - 4x^{\frac{1}{2}} - 1$$

when $x = 3$

$$y = 6\sqrt{3} - 4\sqrt{3} - 1$$

$$y = 2\sqrt{3} - 1$$

$$7 \quad \mathbf{a} \quad = \int (x^2 + 4x + 4) \, dx \\ = \frac{1}{3}x^3 + 2x^2 + 4x + c$$

$$\mathbf{b} \quad = \int \frac{1}{4}x^{-\frac{1}{2}} \, dx \\ = \frac{1}{2}x^{\frac{1}{2}} + c$$

$$9 \quad \mathbf{a} \quad y = \int (2x - 3x^{-2}) \, dx \\ = x^2 + 3x^{-1} + c$$

$$y = 0 \text{ at } x = 1$$

$$\therefore 0 = 1 + 3 + c$$

$$c = -4$$

$$\therefore y = x^2 - 4 + \frac{3}{x}$$

$$\mathbf{b} \quad \frac{d^2y}{dx^2} = 2 + 6x^{-3}$$

$$\therefore x^2 \frac{d^2y}{dx^2} - 2y \\ = x^2(2 + 6x^{-3}) - 2(x^2 - 4 + 3x^{-1}) \\ = 2x^2 + 6x^{-1} - 2x^2 + 8 - 6x^{-1} \\ = 8 \quad [k = 8]$$

$$11 \quad \mathbf{a} \quad f(x) = \int (4x^3 - 8x) \, dx \\ = x^4 - 4x^2 + c \\ (2, -5) \Rightarrow -5 = 16 - 16 + c \\ c = -5$$

$$\therefore f(x) = x^4 - 4x^2 - 5$$

$$\mathbf{b} \quad x^4 - 4x^2 - 5 = 0 \\ (x^2 + 1)(x^2 - 5) = 0 \\ x^2 = -1 \text{ [no sols] or } 5 \\ x = \pm\sqrt{5} \\ \therefore (-\sqrt{5}, 0), (\sqrt{5}, 0)$$

$$8 \quad \mathbf{a} \quad f(x) = \int (3x^2 - 2x - 3) \, dx \\ = x^3 - x^2 - 3x + c$$

$$(-2, 0) \Rightarrow 0 = -8 - 4 + 6 + c$$

$$c = 6$$

$$\therefore f(x) = x^3 - x^2 - 3x + 6$$

$$\mathbf{b} \quad x = 1 \Rightarrow y = 1 - 1 - 3 + 6 = 3 \\ \text{grad} = 3 - 2 - 3 = -2$$

$$\therefore y - 3 = -2(x - 1)$$

$$y - 3 = -2x + 2$$

$$y = 5 - 2x$$

$$10 \quad \mathbf{a} \quad = -\frac{1}{2}x^{-2} + c$$

$$\mathbf{b} \quad = \int \frac{x^2 - 2x + 1}{x^{\frac{1}{2}}} \, dx$$

$$= \int (x^{\frac{3}{2}} - 2x^{\frac{1}{2}} + x^{-\frac{1}{2}}) \, dx$$

$$= \frac{2}{5}x^{\frac{5}{2}} - \frac{4}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + c$$

$$12 \quad \mathbf{a} \quad y = \int (k - x^{-\frac{1}{2}}) \, dx$$

$$y = kx - 2x^{\frac{1}{2}} + c$$

$$(1, -2) \Rightarrow -2 = k - 2 + c$$

$$0 = k + c \quad (1)$$

$$(4, 5) \Rightarrow 5 = 4k - 4 + c$$

$$9 = 4k + c \quad (2)$$

$$(2) - (1) \quad 9 = 3k$$

$$k = 3$$

$$\mathbf{b} \quad \text{grad} = 3 - 1 = 2$$

$$\therefore \text{grad of normal} = -\frac{1}{2}$$

$$\therefore y + 2 = -\frac{1}{2}(x - 1)$$

$$2y + 4 = -x + 1$$

$$x + 2y + 3 = 0$$