

1 Find

$$\int (x^2 + 6\sqrt{x} - 3) \, dx. \quad (3)$$

2 The curve $y = f(x)$ passes through the point $(1, -2)$.

Given that

$$f'(x) = 1 - \frac{6}{x^3},$$

a find an expression for $f(x)$. (4)The point A on the curve $y = f(x)$ has x -coordinate 2.b Show that the normal to the curve $y = f(x)$ at A has the equation

$$16x + 4y - 19 = 0. \quad (5)$$

3 The curve $y = f(x)$ passes through the point $(3, 22)$.

Given that

$$f'(x) = 3x^2 + 2x - 5,$$

a find an expression for $f(x)$. (4)

Given also that

$$g(x) = (x + 3)(x - 1)^2,$$

b show that $g(x) = f(x) + 2$, (3)c sketch the curves $y = f(x)$ and $y = g(x)$ on the same set of axes. (3)

4 Given that

$$y = x^2 - \frac{3}{x^2},$$

find

a $\frac{dy}{dx}$, (2)b $\int y \, dx$. (3)5 The curve C with equation $y = f(x)$ is such that

$$\frac{dy}{dx} = 3x^2 - 4x - 1.$$

Given that the tangent to the curve at the point P with x -coordinate 2 passes through the origin, find an equation for the curve. (7)6 A curve with equation $y = f(x)$ is such that

$$\frac{dy}{dx} = 3\sqrt{x} - \frac{2}{\sqrt{x}}, \quad x > 0.$$

a Find the gradient of the curve at the point where $x = 2$, giving your answer in its simplest form. (2)Given also that the curve passes through the point $(4, 7)$,b find the y -coordinate of the point on the curve where $x = 3$, giving your answer in the form $a\sqrt{3} + b$, where a and b are integers. (6)

7 Find

a $\int (x + 2)^2 dx$, (3)

b $\int \frac{1}{4\sqrt{x}} dx$. (3)

8 The curve C has the equation $y = f(x)$ and crosses the x -axis at the point $P(-2, 0)$.

Given that

$$f'(x) = 3x^2 - 2x - 3,$$

a find an expression for $f(x)$, (4)

b show that the tangent to the curve at the point where $x = 1$ has the equation

$$y = 5 - 2x. \quad (3)$$

9 Given that

$$\frac{dy}{dx} = 2x - \frac{3}{x^2}, \quad x \neq 0,$$

and that $y = 0$ at $x = 1$,

a find an expression for y in terms of x , (4)

b show that for all non-zero values of x

$$x^2 \frac{d^2y}{dx^2} - 2y = k,$$

where k is a constant to be found. (4)

10 Integrate with respect to x

a $\frac{1}{x^3}$, (2)

b $\frac{(x-1)^2}{\sqrt{x}}$. (5)

11 The curve $y = f(x)$ passes through the point $(2, -5)$.

Given that

$$f'(x) = 4x^3 - 8x,$$

a find an expression for $f(x)$, (4)

b find the coordinates of the points where the curve crosses the x -axis. (4)

12 The curve C with equation $y = f(x)$ is such that

$$\frac{dy}{dx} = k - x^{-\frac{1}{2}}, \quad x > 0,$$

where k is a constant.

Given that C passes through the points $(1, -2)$ and $(4, 5)$,

a find the value of k , (5)

b show that the normal to C at the point $(1, -2)$ has the equation

$$x + 2y + 3 = 0. \quad (4)$$