

- 1 Write down the first five terms of the sequences with n th terms, u_n , given for $n \geq 1$ by
- a** $u_n = 4n + 5$ **b** $u_n = (n + 1)^2$ **c** $u_n = 2^n$ **d** $u_n = \frac{n}{n+1}$
- e** $u_n = n^3 - 2n$ **f** $u_n = 1 - \frac{1}{3}n$ **g** $u_n = 1 - \frac{1}{2n}$ **h** $u_n = 32 \times (\frac{1}{2})^n$
- 2 The n th term of each of the following sequences is given by $u_n = an + b$, for $n \geq 1$. Find the values of the constants a and b in each case.
- a** 4, 7, 10, 13, 16, ... **b** 0, 7, 14, 21, 28, ... **c** 16, 14, 12, 10, 8, ...
- d** 0.4, 1.7, 3.0, 4.3, 5.6, ... **e** 100, 83, 66, 49, 32, ... **f** -13, -5, 3, 11, 19, ...
- 3 Find a possible expression for the n th term of each of the following sequences.
- a** 1, 6, 11, 16, 21, ... **b** 3, 9, 27, 81, 243, ... **c** 2, 8, 18, 32, 50, ...
- d** $\frac{1}{2}$, 1, 2, 4, 8, ... **e** 22, 11, 0, -11, -22, ... **f** 0, 1, 8, 27, 64, ...
- g** 4, 7, 12, 19, 28, ... **h** $\frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{4}{9}, \frac{5}{11}, \dots$ **i** 1, 3, 7, 15, 31, ...
- 4 The n th term of a sequence, u_n , is given by
- $$u_n = c + 3^{n-2}.$$
- Given that $u_3 = 11$,
- a** find the value of the constant c ,
- b** find the value of u_6 .
- 5 The n th term of a sequence, u_n , is given by
- $$u_n = n(2n + k).$$
- Given that $u_6 = 2u_4 - 2$,
- a** find the value of the constant k ,
- b** prove that for all values of n , $u_n - u_{n-1} = 4n + 3$.
- 6 The n th term of a sequence, u_n , is given by
- $$u_n = k^n - 3.$$
- Given that $u_1 + u_2 = 0$,
- a** find the two possible values of the constant k .
- b** For each value of k found in part **a**, find the corresponding value of u_5 .
- 7 Write down the first four terms of each sequence.
- a** $u_n = u_{n-1} + 4$, $n > 1$, $u_1 = 3$ **b** $u_n = 3u_{n-1} + 1$, $n > 1$, $u_1 = 2$
- c** $u_{n+1} = 2u_n + 5$, $n > 0$, $u_1 = -2$ **d** $u_n = 7 - u_{n-1}$, $n \geq 2$, $u_1 = 5$
- e** $u_n = 2(5 - 2u_{n-1})$, $n > 1$, $u_1 = -1$ **f** $u_n = \frac{1}{10}(u_{n-1} + 20)$, $n \geq 2$, $u_1 = 10$
- g** $u_{n+1} = 1 - \frac{1}{3}u_n$, $n \geq 1$, $u_1 = 6$ **h** $u_{n+1} = \frac{1}{2+u_n}$, $n \geq 1$, $u_1 = 0$

- 8** In each case, write down a recurrence relation that would produce the given sequence.
- a** 5, 9, 13, 17, 21, ... **b** 1, 3, 9, 27, 81, ... **c** 62, 44, 26, 8, -10, ...
d 120, 60, 30, 15, 7.5, ... **e** 4, 9, 19, 39, 79, ... **f** 1, 3, 11, 43, 171, ...
- 9** Given that the following sequences can be defined by recurrence relations of the form $u_n = au_{n-1} + b$, $n > 1$, find the values of the constants a and b for each sequence.
- a** -4, -3, -1, 3, 11, ... **b** 0, 8, 4, 6, 5, ... **c** $7\frac{3}{4}$, $5\frac{1}{2}$, 4, 3, $2\frac{1}{3}$, ...
- 10** For each of the following sequences, find expressions for u_2 and u_3 in terms of the constant k .
- a** $u_n = 4u_{n-1} + 3k$, $n > 1$, $u_1 = 1$ **b** $u_{n+1} = ku_n + 5$, $n > 0$, $u_1 = 2$
c $u_n = 4u_{n-1} - k$, $n > 1$, $u_1 = k$ **d** $u_n = 2 - ku_{n-1}$, $n \geq 2$, $u_1 = -1$
e $u_{n+1} = \frac{u_n}{k}$, $n \geq 1$, $u_1 = 4$ **f** $u_{n+1} = \sqrt[3]{61k^3 + u_n^3}$, $n > 0$, $u_1 = k\sqrt[3]{3}$
- 11** A sequence is given by the recurrence relation
- $$u_n = \frac{1}{2}(k + 3u_{n-1}), \quad n > 1, \quad u_1 = 2.$$
- a** Find an expression for u_3 in terms of the constant k .
Given that $u_3 = 7$,
b find the value of k and the value of u_4 .
- 12** For the sequences given by the following recurrence relations, find u_4 and u_1 .
- a** $u_n = 3u_{n-1} - 2$, $n > 1$, $u_3 = 10$ **b** $u_{n+1} = \frac{3}{4}u_n + 2$, $n > 0$, $u_3 = 5$
c $u_{n+1} = 0.2(1 - u_n)$, $n > 0$, $u_3 = -0.2$ **d** $u_n = \frac{1}{2}\sqrt{u_{n-1}}$, $n > 1$, $u_3 = 1$
- 13** A sequence is defined by
- $$u_{n+1} = u_n + c, \quad n \geq 1, \quad u_1 = 2,$$
- where c is a constant. Given that $u_5 = 30$, find
- a** the value of c ,
b an expression for u_n in terms of n .
- 14** The terms of a sequence u_1, u_2, u_3, \dots are given by
- $$u_n = 3(u_{n-1} - k), \quad n > 1,$$
- where k is a constant. Given that $u_1 = -4$,
- a** find expressions for u_2 and u_3 in terms of k .
Given also that $u_3 = 7u_2 + 3$, find
b the value of k ,
c the value of u_4 .
- 15** A sequence of terms $\{t_n\}$ is defined, for $n > 1$, by the recurrence relation
- $$t_n = kt_{n-1} + 2,$$
- where k is a constant. Given that $t_1 = 1.5$,
- a** find expressions for t_2 and t_3 in terms of k .
Given also that $t_3 = 12$,
b find the possible values of k .