**1** **a** 9, 13, 17, 21, 25 **b** 4, 9, 16, 25, 36 **c** 2, 4, 8, 16, 32 **d** $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}$

**e** –1, 4, 21, 56, 115 **f** $\frac{2}{3}, \frac{1}{3}, 0, -\frac{1}{3}, -\frac{2}{3}$ **g** $\frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \frac{9}{10}$ **h** 16, 8, 4, 2, 1

**2** **a** $u_n = 3n + 1$
$a = 3,\ b = 1$

**b** $u_n = 7n - 7$
$a = 7,\ b = -7$

**c** $u_n = 18 - 2n$
$a = -2,\ b = 18$

**d** $u_n = 1.3n - 0.9$
$a = 1.3,\ b = -0.9$

**e** $u_n = 117 - 17n$
$a = -17,\ b = 117$

**f** $u_n = 8n - 21$
$a = 8,\ b = -21$

**3** possible answers are

**a** $5n - 4$ **b** $3^n$ **c** $2n^2$

**d** $\frac{1}{4} \times 2^n$ **e** $33 - 11n$ **f** $(n - 1)^3$

**g** $n^2 + 3$ **h** $\frac{n}{2n+1}$ **i** $2^n - 1$

**4** **a** $u_3 = c + 3 = 11 \quad \therefore\ c = 8$

**b** $u_6 = 8 + 3^4 = 89$

**5** **a** $u_4 = 4(8 + k) = 32 + 4k$
$u_6 = 6(12 + k) = 72 + 6k$
$\therefore\ 72 + 6k = 2(32 + 4k) - 2$
$72 + 6k = 62 + 8k$
$k = 5$

**b** $u_n = n(2n + 5) = 2n^2 + 5n$
$u_{n-1} = (n - 1)[2(n - 1) + 5] = (n - 1)(2n + 3) = 2n^2 + n - 3$
$\therefore\ u_n - u_{n-1} = (2n^2 + 5n) - (2n^2 + n - 3) = 4n + 3$

**6** **a** $u_1 = k - 3$
$u_2 = k^2 - 3$
$\therefore\ k - 3 + k^2 - 3 = 0$
$k^2 + k - 6 = 0$
$(k + 3)(k - 2) = 0$
$k = -3$ or $2$

**b** $k = -3 \quad \Rightarrow \quad u_5 = (-3)^5 - 3 = -243 - 3 = -246$
$k = 2 \quad \Rightarrow \quad u_5 = 2^5 - 3 = 32 - 3 = 29$

**7** **a** 3, 7, 11, 15 **b** 2, 7, 22, 67

**c** –2, 1, 7, 19 **d** 5, 2, 5, 2

**e** –1, 14, –46, 194 **f** 10, 3, 2.3, 2.23

**g** $6, -1, 1\frac{1}{3}, \frac{5}{9}$ **h** $0, \frac{1}{2}, \frac{2}{5}, \frac{5}{12}$

**8** possible answers are

**a** $u_{n+1} = u_n + 4,\ u_1 = 5$ **b** $u_{n+1} = 3u_n,\ u_1 = 1$ **c** $u_{n+1} = u_n - 18,\ u_1 = 62$

**d** $u_{n+1} = \frac{1}{2}u_n,\ u_1 = 120$ **e** $u_{n+1} = 2u_n + 1,\ u_1 = 4$ **f** $u_{n+1} = 4u_n - 1,\ u_1 = 1$

**1** **a** $a + 2d = -10 \quad (1)$

$\frac{8}{2}(2a + 7d) = 16 \Rightarrow 2a + 7d = 4$

$2 \times (1) \qquad \Rightarrow 2a + 4d = -20$

subtracting, $3d = 24$

$d = 8$

sub. $a = -26$

**b** $-26 + 8(n - 1) > 300$

$n > 41\frac{3}{4} \quad \therefore$ smallest $n = 42$

**2** **a** $a + 2d = \frac{5}{6}$

$a + 6d = 2\frac{1}{3}$

subtracting, $4d = 1\frac{1}{2}$

$d = \frac{3}{8}$

sub. $a = \frac{1}{12}$

**b** $S_n = \frac{n}{2}[\frac{1}{6} + \frac{3}{8}(n - 1)]$

$= \frac{1}{48}n[4 + 9(n - 1)]$

$= \frac{1}{48}n(9n - 5) \qquad [\, k = \frac{1}{48}\,]$

**3** **a** $\frac{9}{2}(2a + 8d) = 126$

$9(a + 4d) = 126$

$a + 4d = 14$

**b** $\frac{15}{2}(2a + 14d) = 277.5$

$a + 7d = 18.5$

subtracting, $3d = 4.5$

$d = 1.5$

sub. $a = 8$

**c** $S_{32} = \frac{32}{2}[16 + (31 \times 1.5)] = 1000$

**4** **a** $(5k + 3) - (7k - 1) = (4k + 1) - (5k + 3)$

$-2k + 4 = -k - 2$

$k = 6$

**b** given terms = 41, 33, 25

$d = -8$

smallest +ve term $= 25 + (3 \times -8) = 1$

**c** consider series of +ve terms in reverse

$a = 1,\ d = 8$

$S_r = \frac{r}{2}[2 + 8(r - 1)] = r(4r - 3)$

**5** **a** AP: $a = 4,\ l = 120,\ n = 30$

$S_{30} = \frac{30}{2}(4 + 120) = 1860$

**b** **i** $= \sum_{r=1}^{30} 4r + 30 = 1890$

**ii** $= 2 \times \sum_{r=1}^{30} 4r - (30 \times 5)$

$= (2 \times 1860) - 150 = 3570$

**6** **a** $500 + (7 \times 40) = £780$

**b** AP: $a = 500,\ d = 40$

$S_n = \frac{n}{2}[1000 + 40(n - 1)] = 20n(n + 24)$

**c** AP: $a = 400,\ d = 60$

$S_n = \frac{n}{2}[800 + 60(n - 1)] = 10n(3n + 37)$

$\therefore\ 20n(n + 24) = 10n(3n + 37)$

$n \neq 0 \ \therefore\ 2(n + 24) = (3n + 37)$

$n = 11 \ \therefore$ 11 years

**7** **a** $S_n = 2 + 4 + 6 + \ldots + (2n - 2) + 2n$

write in reverse

$S_n = 2n + (2n - 2) + \ldots + 6 + 4 + 2$

adding, $2S_n = n \times (2n + 2)$

$S_n = n(n + 1)$

**b** integers 200 to 800, AP: $n = 601$

$S_{601} = \frac{601}{2}(200 + 800) = 300\,500$

integers 200 to 800 divisible by 4

AP: $a = 200,\ l = 800$

$200 + 4(n - 1) = 800 \Rightarrow n = 151$

$S_{151} = \frac{151}{2}(200 + 800) = 75\,500$

required sum $= 300\,500 - 75\,500$

$= 225\,000$

**8** **a** $S_n = \frac{1}{2}n[2a + (n - 1)d]$

**b** $S_2 = \frac{2}{2}(2a + d) = 2a + d$

$S_6 = \frac{6}{2}(2a + 5d) = 6a + 15d$

$S_8 = \frac{8}{2}(2a + 7d) = 8a + 28d$

$2(S_6 - S_2) = 2[(6a + 15d) - (2a + d)]$

$= 2(4a + 14d)$

$= 8a + 28d = S_8$

**c** for +ve terms $\quad 40 - 3(n - 1) > 0$

$n < \frac{43}{3} \ \therefore$ 14 terms

$\therefore\ S_{14} = \frac{14}{2}[80 + (13 \times -3)] = 287$

**9** **a** **i** $u_4 - u_1 = x + 3$

$u_7 = u_4 + (x + 3) = 3x + 6$

**ii** $3d = x + 3$

$d = \frac{1}{3}x + 1$

**iii** $S_{10} = \frac{10}{2}[2x + 9(\frac{1}{3}x + 1)]$

$= 5[2x + 3x + 9] = 25x + 45$

**b** $x + 19(\frac{1}{3}x + 1) = 52$

$3x + 19x + 57 = 156$

$x = \frac{99}{22} = \frac{9}{2}$ or $4\frac{1}{2}$

**10** $S_{20} = \frac{20}{2}(2a + 19d) = 20a + 190d$

$S_{30} = \frac{30}{2}(2a + 29d) = 30a + 435d$

$S_{30} - S_{20} = 10a + 245d$

$\therefore\ 20a + 190d = 10a + 245d$

$10a = 55d$

$2a = 11d$

$\therefore\ a : d = 11 : 2$

**11** **a** $S_6 = 12(16 - 6) = 120$

$S_5 = 10(16 - 5) = 110$

$u_6 = S_6 - S_5 = 10$

**b** $S_n = 2n(16 - n) = 32n - 2n^2$

$S_{n-1} = 2(n - 1)[16 - (n - 1)]$

$= 2(n - 1)(17 - n)$

$= -2n^2 + 36n - 34$

$u_n = S_n - S_{n-1}$

$= (32n - 2n^2) - (-2n^2 + 36n - 34)$

$= 34 - 4n$

**c** $u_{n-1} = 34 - 4(n - 1) = 38 - 4n$

$u_n - u_{n-1} = (34 - 4n) - (38 - 4n) = -4$

$u_n - u_{n-1}$ constant $\therefore$ arithmetic series

**12** **a** **i** $2400 + (5 \times 250) = 3650$

**ii** AP: $a = 2400$, $d = 250$

$S_{10} = \frac{10}{2}[4800 + (9 \times 250)]$

$= 35\,250$

**b** AP: $a = 2400$, $d = C$

$\frac{10}{2}[4800 + (9 \times C)] = 40\,000$

$C = \frac{3200}{9} = 356$ (nearest unit)

**13** **a** let common difference be $d$

$S_r = a + (a + d) + (a + 2d) + \ldots + (l - 2d) + (l - d) + l$

write in reverse

$S_r = l + (l - d) + (l - 2d) + \ldots + (a + 2d) + (a + d) + a$

adding, $2S_r = r \times (a + l)$

$S_r = \frac{1}{2}r(a + l)$

**b** $n = 18$, $l = 68$, $S_{18} = 153$

$\therefore\ 153 = \frac{18}{2}(a + 68)$

$a = 17 - 68 = -51$