

- 1** **a** $a + d = 40$, $a + 4d = 121$
 subtracting, $3d = 81$
 $d = 27$
 sub. $a = 13$
b $S_{25} = \frac{25}{2} [26 + (24 \times 27)] = 8425$
- 2** **a** 3, 7, 11, 15, 19
b AP: $a = 3$, $d = 4$, $n = 20$
 $S_{20} = \frac{20}{2} [6 + (19 \times 4)]$
 $= 820$
- 3** **a** $(2t - 5) - t = 8.6 - (2t - 5)$
 $t = 6.2$
b $u_1 = 6.2$, $u_2 = 12.4 - 5 = 7.4$
 $a = 6.2$, $d = 7.4 - 6.2 = 1.2$
 $u_{16} = 6.2 + (15 \times 1.2) = 24.2$
c $S_{20} = \frac{20}{2} [12.4 + (19 \times 1.2)] = 352$
- 4** **a** $S_n = \frac{1}{2}n(n + 1)$
b $= S_{400} - S_{199}$
 $= \frac{1}{2} \times 400 \times 401 - \frac{1}{2} \times 199 \times 200$
 $= 80\,200 - 19\,900 = 60\,300$
c $\frac{1}{2}N(N + 1) = 4950$
 $N^2 + N - 9900 = 0$
 $(N + 100)(N - 99) = 0$
 $N > 0 \therefore N = 99$
- 5** **a** $u_2 = k + 1$
 $u_3 = k + (k + 1)^2 = k^2 + 3k + 1$
b $k^2 + 3k + 1 = 1$
 $k(k + 3) = 0$
 $k \neq 0 \therefore k = -3$
c $u_{25} = 1$
 $u_1 = 1 \Rightarrow u_3 = 1$
 $\therefore u_n = 1$ for all odd values of n
- 6** **a** AP: $a = 3$, $d = 3$
 $500 \div 3 = 166\frac{2}{3} \therefore n = 166$
 $S_{166} = \frac{166}{2} [6 + (165 \times 3)]$
 $= 41\,583$
b AP: $a = 14$, $l = 99$, $n = 18$
 $S_{18} = \frac{18}{2} (14 + 99)$
 $= 1017$
- 7** **a** $S_n = a + (a + d) + (a + 2d) + \dots$
 $+ [a + (n - 2)d] + [a + (n - 1)d]$
 write in reverse
 $S_n = [a + (n - 1)d] + [a + (n - 2)d] + \dots$
 $+ (a + 2d) + (a + d) + a$
 adding $2S_n = n \times \{a + [a + (n - 1)d]\}$
 $S_n = \frac{1}{2}n[2a + (n - 1)d]$
b $S_{26} = \frac{26}{2} [-2 + (25 \times 6)] = 1924$
 $S_{27} = \frac{27}{2} [-2 + (26 \times 6)] = 2079$
 \therefore largest $n = 26$
- 8** $t_2 = 4 + 2k$
 $t_3 = 4 - k(4 + 2k)$
 $\therefore 4 - 4k - 2k^2 = 3$
 $2k^2 + 4k - 1 = 0$
 $k = \frac{-4 \pm \sqrt{16 + 8}}{4} = \frac{-4 \pm 2\sqrt{6}}{4}$
 $k > 0 \therefore k = -1 + \frac{1}{2}\sqrt{6}$
- 9** **a** $= 6 + (19 \times 3) = 63$
b $S_n = \frac{n}{2} [12 + 3(n - 1)] = 270$
 $\therefore n(3n + 9) = 540$
 $n^2 + 3n - 180 = 0$
 $(n + 15)(n - 12) = 0$
 $n > 0 \therefore n = 12$
- 10** **a** $= 3 \times 570 = 1710$
b $= 570 + (2 \times 30) = 630$
c $= 570 + (\frac{1}{2} \times 30 \times 31) = 1035$

- 11 a** 2 years = 8×3 months
total = $3 \times S_8$ [AP: $a = 40, d = 2$]
 $= 3 \times \frac{8}{2} [80 + (7 \times 2)]$
 $= 3 \times 376 = \text{£}1128$
- b** n years = $4n \times 3$ months
total = $3 \times S_{4n}$
 $= 3 \times \frac{4n}{2} \{80 + [(4n - 1) \times 2]\}$
 $= 6n(80 + 8n - 2)$
 $= 12n(4n + 39)$
- 12** AP: $a = 80, d = -3, n = 45$
 $S_{45} = \frac{45}{2} [160 + (44 \times -3)] = 630$
- 13 a** $a + 2d = 298, a + 7d = 263$
subtracting, $5d = -35$
 $d = -7$
- b** sub. $a = 312$
 $312 - 7(n - 1) > 0$
 $n < \frac{319}{7} \therefore 45$ positive terms
- c** max S_n when $n = 45$
 $S_{45} = \frac{45}{2} [624 + (44 \times -7)] = 7110$
- 14 a** AP: $a = 10, d = 6$
 $S_n = \frac{n}{2} [20 + 6(n - 1)]$
 $= n(3n + 7)$
- b** $S_{2n} = 2n[(3 \times 2n) + 7]$
 $= 12n^2 + 14n$
required sum = $S_{2n} - S_n$
 $= (12n^2 + 14n) - (3n^2 + 7n)$
 $= 9n^2 + 7n = n(9n + 7)$
- 15 a** $u_2 = k^2 - 2, u_4 = k^4 - 4$
 $\therefore k^2 - 2 + k^4 - 4 = 6$
 $k^4 + k^2 - 12 = 0$
 $(k^2 + 4)(k^2 - 3) = 0$
 $k^2 = -4$ [no solutions] or 3
 $k > 0 \therefore k = \sqrt{3}$
- b** $u_1 = \sqrt{3} - 1$
 $u_3 = (\sqrt{3})^3 - 3 = 3(\sqrt{3} - 1) = 3u_1$
- 16 a** $(4k - 2) - (k + 4) = (k^2 - 2) - (4k - 2)$
 $3k - 6 = k^2 - 4k$
 $k^2 - 7k + 6 = 0$
- b** $(k - 1)(k - 6) = 0$
 $k = 1$ or 6
 $d = 3k - 6$
 $d > 0 \therefore k = 6$
 $a = 10, d = 12$
 $u_{15} = 10 + (14 \times 12) = 178$