

1 a

$$\begin{array}{r}
 x^2 + x - 2 \\
 x+1 \overline{) x^3 + 2x^2 - x - 2} \\
 \underline{x^3 + x^2} \phantom{- x - 2} \\
 x^2 - x \phantom{- 2} \\
 \underline{x^2 + x} \phantom{- 2} \\
 -2x - 2 \\
 \underline{-2x - 2} \\
 0
 \end{array}$$

quotient:  $x^2 + x - 2$ 

b

$$\begin{array}{r}
 x^2 + 4x - 1 \\
 x-2 \overline{) x^3 + 2x^2 - 9x + 2} \\
 \underline{x^3 - 2x^2} \phantom{- 9x + 2} \\
 4x^2 - 9x \phantom{+ 2} \\
 \underline{4x^2 - 8x} \phantom{+ 2} \\
 -x + 2 \\
 \underline{-x + 2} \\
 0
 \end{array}$$

quotient:  $x^2 + 4x - 1$ 

c

$$\begin{array}{r}
 x^2 - x + 5 \\
 x+4 \overline{) x^3 + 3x^2 + x + 20} \\
 \underline{x^3 + 4x^2} \phantom{+ x + 20} \\
 -x^2 + x \phantom{+ 20} \\
 \underline{-x^2 - 4x} \phantom{+ 20} \\
 5x + 20 \\
 \underline{5x + 20} \\
 0
 \end{array}$$

quotient:  $x^2 - x + 5$ 

d

$$\begin{array}{r}
 2x^2 + x - 3 \\
 x-1 \overline{) 2x^3 - x^2 - 4x + 3} \\
 \underline{2x^3 - 2x^2} \phantom{- 4x + 3} \\
 x^2 - 4x \phantom{+ 3} \\
 \underline{x^2 - x} \phantom{+ 3} \\
 -3x + 3 \\
 \underline{-3x + 3} \\
 0
 \end{array}$$

quotient:  $2x^2 + x - 3$ 

e

$$\begin{array}{r}
 6x^2 + 11x - 18 \\
 x-5 \overline{) 6x^3 - 19x^2 - 73x + 90} \\
 \underline{6x^3 - 30x^2} \phantom{- 73x + 90} \\
 11x^2 - 73x \phantom{+ 90} \\
 \underline{11x^2 - 55x} \phantom{+ 90} \\
 -18x + 90 \\
 \underline{-18x + 90} \\
 0
 \end{array}$$

quotient:  $6x^2 + 11x - 18$ 

f

$$\begin{array}{r}
 -x^2 + 7x - 4 \\
 x+2 \overline{) -x^3 + 5x^2 + 10x - 8} \\
 \underline{-x^3 - 2x^2} \phantom{+ 10x - 8} \\
 7x^2 + 10x \phantom{- 8} \\
 \underline{7x^2 + 14x} \phantom{- 8} \\
 -4x - 8 \\
 \underline{-4x - 8} \\
 0
 \end{array}$$

quotient:  $-x^2 + 7x - 4$ 

g

$$\begin{array}{r}
 x^2 - 3x + 7 \\
 x+3 \overline{) x^3 + 0x^2 - 2x + 21} \\
 \underline{x^3 + 3x^2} \phantom{- 2x + 21} \\
 -3x^2 - 2x \phantom{+ 21} \\
 \underline{-3x^2 - 9x} \phantom{+ 21} \\
 7x + 21 \\
 \underline{7x + 21} \\
 0
 \end{array}$$

quotient:  $x^2 - 3x + 7$ 

h

$$\begin{array}{r}
 3x^2 - 2x + 12 \\
 x+6 \overline{) 3x^3 + 16x^2 + 0x + 72} \\
 \underline{3x^3 + 18x^2} \phantom{+ 0x + 72} \\
 -2x^2 + 0x \phantom{+ 72} \\
 \underline{-2x^2 - 12x} \phantom{+ 72} \\
 12x + 72 \\
 \underline{12x + 72} \\
 0
 \end{array}$$

quotient:  $3x^2 - 2x + 12$

$$\begin{array}{r}
 2 \quad \mathbf{a} \quad \frac{x^2 + 3x + 2}{x + 5} \overline{) x^3 + 8x^2 + 17x + 16} \\
 \underline{x^3 + 5x^2} \phantom{+ 17x + 16} \\
 3x^2 + 17x \phantom{+ 16} \\
 \underline{3x^2 + 15x} \phantom{+ 16} \\
 2x + 16 \\
 \underline{2x + 10} \\
 6
 \end{array}$$

quotient:  $x^2 + 3x + 2$  remainder: 6

$$\begin{array}{r}
 \mathbf{b} \quad \frac{x^2 - 8x + 5}{x - 7} \overline{) x^3 - 15x^2 + 61x - 48} \\
 \underline{x^3 - 7x^2} \phantom{+ 61x - 48} \\
 -8x^2 + 61x \phantom{- 48} \\
 \underline{-8x^2 + 56x} \phantom{- 48} \\
 5x - 48 \\
 \underline{5x - 35} \\
 -13
 \end{array}$$

quotient:  $x^2 - 8x + 5$  remainder: -13

$$\begin{array}{r}
 \mathbf{c} \quad \frac{3x^2 - 2x + 4}{x + 2} \overline{) 3x^3 + 4x^2 + 0x + 7} \\
 \underline{3x^3 + 6x^2} \phantom{+ 0x + 7} \\
 -2x^2 + 0x \phantom{+ 7} \\
 \underline{-2x^2 - 4x} \phantom{+ 7} \\
 4x + 7 \\
 \underline{4x + 8} \\
 -1
 \end{array}$$

quotient:  $3x^2 - 2x + 4$  remainder: -1

$$\begin{array}{r}
 \mathbf{d} \quad \frac{-x^2 + 3x - 9}{x + 8} \overline{) -x^3 - 5x^2 + 15x - 50} \\
 \underline{-x^3 - 8x^2} \phantom{+ 15x - 50} \\
 3x^2 + 15x \phantom{- 50} \\
 \underline{3x^2 + 24x} \phantom{- 50} \\
 -9x - 50 \\
 \underline{-9x - 72} \\
 22
 \end{array}$$

quotient:  $-x^2 + 3x - 9$  remainder: 22

$$\begin{array}{r}
 \mathbf{e} \quad \frac{4x^2 + 14x + 26}{x - 3} \overline{) 4x^3 + 2x^2 - 16x + 3} \\
 \underline{4x^3 - 12x^2} \phantom{- 16x + 3} \\
 14x^2 - 16x \phantom{+ 3} \\
 \underline{14x^2 - 42x} \phantom{+ 3} \\
 26x + 3 \\
 \underline{26x - 78} \\
 81
 \end{array}$$

quotient:  $4x^2 + 14x + 26$  remainder: 81

$$\begin{array}{r}
 \mathbf{f} \quad \frac{-6x^2 - 10x + 20}{x + 2} \overline{) -6x^3 - 22x^2 + 0x + 1} \\
 \underline{-6x^3 - 12x^2} \phantom{+ 0x + 1} \\
 -10x^2 + 0x \phantom{+ 1} \\
 \underline{-10x^2 - 20x} \phantom{+ 1} \\
 20x + 1 \\
 \underline{20x + 40} \\
 -39
 \end{array}$$

quotient:  $-6x^2 - 10x + 20$  remainder: -39

**3 a** let  $f(x) \equiv x^3 + 2x^2 - 2x - 1$   
 $f(1) = 1 + 2 - 2 - 1 = 0$   
 $\therefore (x - 1)$  is a factor

**c** let  $f(x) \equiv x^3 - x^2 - 14x + 27$   
 $f(3) = 27 - 9 - 42 + 27 = 3$   
 $\therefore (x - 3)$  is not a factor

**e** let  $f(x) \equiv 2x^3 - 5x^2 + 7x - 14$   
 $f(-\frac{1}{2}) = -\frac{1}{4} - \frac{5}{4} - \frac{7}{2} - 14 = -19$   
 $\therefore (2x + 1)$  is not a factor

**b** let  $f(x) \equiv x^3 - 5x^2 - 9x + 2$   
 $f(-2) = -8 - 20 + 18 + 2 = -8$   
 $\therefore (x + 2)$  is not a factor

**d** let  $f(x) \equiv 2x^3 + 13x^2 + 2x - 24$   
 $f(-6) = -432 + 468 - 12 - 24 = 0$   
 $\therefore (x + 6)$  is a factor

**f** let  $f(x) \equiv 2 - 17x + 25x^2 - 6x^3$   
 $f(\frac{2}{3}) = 2 - \frac{34}{3} + \frac{100}{9} - \frac{16}{9} = 0$   
 $\therefore (3x - 2)$  is a factor

4 a  $f(1) = 1 - 2 - 11 + 12 = 0$   
 $\therefore (x - 1)$  is a factor of  $f(x)$

b

$$\begin{array}{r} x^2 + x - 12 \\ x-1 \overline{) x^3 - 2x^2 - 11x + 12} \\ \underline{x^3 - x^2} \phantom{+ 12} \\ -x^2 - 11x \phantom{+ 12} \\ \underline{-x^2 + x} \phantom{+ 12} \\ -12x + 12 \\ \underline{-12x + 12} \\ 0 \end{array}$$

$$\begin{aligned} \therefore f(x) &\equiv (x-1)(x^2 + x - 12) \\ &\equiv (x-1)(x+3)(x-4) \end{aligned}$$

5  $g(-3) = -54 + 9 + 39 + 6 = 0$   
 $\therefore (x + 3)$  is a factor of  $g(x)$

$$\begin{array}{r} 2x^2 - 5x + 2 \\ x+3 \overline{) 2x^3 + x^2 - 13x + 6} \\ \underline{2x^3 + 6x^2} \phantom{+ 6} \\ -5x^2 - 13x \phantom{+ 6} \\ \underline{-5x^2 - 15x} \phantom{+ 6} \\ 2x + 6 \\ \underline{2x + 6} \\ 0 \end{array}$$

$$\begin{aligned} \therefore g(x) &\equiv (x+3)(2x^2 - 5x + 2) \\ &\equiv (x+3)(2x-1)(x-2) \end{aligned}$$

$$\begin{aligned} g(x) = 0 &\Rightarrow (x+3)(2x-1)(x-2) = 0 \\ x &= -3, \frac{1}{2} \text{ or } 2 \end{aligned}$$

6  $f(4) = 0 \therefore (x - 4)$  is a factor of  $f(x)$

$$\begin{array}{r} 6x^2 + 17x - 3 \\ x-4 \overline{) 6x^3 - 7x^2 - 71x + 12} \\ \underline{6x^3 - 24x^2} \phantom{+ 12} \\ 17x^2 - 71x \phantom{+ 12} \\ \underline{17x^2 - 68x} \phantom{+ 12} \\ -3x + 12 \\ \underline{-3x + 12} \\ 0 \end{array}$$

$$\begin{aligned} \therefore f(x) &\equiv (x-4)(6x^2 + 17x - 3) \\ &\equiv (x-4)(6x-1)(x+3) \end{aligned}$$

$$\begin{aligned} f(x) = 0 &\Rightarrow (x-4)(6x-1)(x+3) = 0 \\ x &= -3, \frac{1}{6} \text{ or } 4 \end{aligned}$$

7 a  $g(-2) = 0 \therefore (x + 2)$  is a factor of  $g(x)$

$$\begin{array}{r} x^2 + 5x - 3 \\ x+2 \overline{) x^3 + 7x^2 + 7x - 6} \\ \underline{x^3 + 2x^2} \phantom{+ 6} \\ 5x^2 + 7x \phantom{+ 6} \\ \underline{5x^2 + 10x} \phantom{+ 6} \\ -3x - 6 \\ \underline{-3x - 6} \\ 0 \end{array}$$

$$\therefore g(x) \equiv (x+2)(x^2 + 5x - 3)$$

b other solutions given by  $x^2 + 5x - 3 = 0$

$$x = \frac{-5 \pm \sqrt{25+12}}{2} = \frac{-5 \pm \sqrt{37}}{2}$$

$$x = -5.54 \text{ or } 0.54$$

8 a  $f(1) = 1 + 2 - 11 - 12 = -20$   
 $f(2) = 8 + 8 - 22 - 12 = -18$   
 $f(-1) = -1 + 2 + 11 - 12 = 0$   
 $f(-2) = -8 + 8 + 22 - 12 = 10$

b  $(x + 1)$  is a factor of  $f(x)$ 

$$\begin{array}{r} x^2 + x - 12 \\ x+1 \overline{) x^3 + 2x^2 - 11x - 12} \\ \underline{x^3 + x^2} \phantom{- 12} \\ x^2 - 11x \phantom{- 12} \\ \underline{x^2 + x} \phantom{- 12} \\ -12x - 12 \\ \underline{-12x - 12} \\ 0 \end{array}$$

$$\begin{aligned} \therefore f(x) &= (x+1)(x^2 + x - 12) \\ &= (x+1)(x+4)(x-3) \end{aligned}$$

- 9 a let  $f(x) = x^3 - 2x^2 - 5x + 6$     b let  $f(x) = x^3 + x^2 - 5x - 2$     c let  $f(x) = 20 + 11x - 8x^2 + x^3$   
 $f(1) = 0$      $f(1) = -5, f(2) = 0$      $f(1) = 24, f(2) = 18, f(-1) = 0$   
 $\therefore (x - 1)$  is a factor     $\therefore (x - 2)$  is a factor     $\therefore (x + 1)$  is a factor

$$\begin{array}{r} x^2 - x - 6 \\ x-1 \overline{) x^3 - 2x^2 - 5x + 6} \\ \underline{x^3 - x^2} \phantom{+ 6} \\ -x^2 - 5x \phantom{+ 6} \\ \underline{-x^2 + x} \phantom{+ 6} \\ -6x + 6 \\ \underline{-6x + 6} \\ 0 \end{array}$$

$$\begin{aligned} \therefore \\ f(x) &= (x-1)(x^2 - x - 6) \\ &= (x-1)(x+2)(x-3) \end{aligned}$$

$$\begin{array}{r} x^2 + 3x + 1 \\ x-2 \overline{) x^3 + x^2 - 5x - 2} \\ \underline{x^3 - 2x^2} \phantom{- 2} \\ 3x^2 - 5x \phantom{- 2} \\ \underline{3x^2 - 6x} \phantom{- 2} \\ x - 2 \\ \underline{x - 2} \\ 0 \end{array}$$

$$\begin{aligned} \therefore \\ f(x) &= (x-2)(x^2 + 3x + 1) \end{aligned}$$

$$\begin{array}{r} x^2 - 9x + 20 \\ x+1 \overline{) x^3 - 8x^2 + 11x + 20} \\ \underline{x^3 + x^2} \phantom{+ 20} \\ -9x^2 + 11x \phantom{+ 20} \\ \underline{-9x^2 - 9x} \phantom{+ 20} \\ 20x + 20 \\ \underline{20x + 20} \\ 0 \end{array}$$

$$\begin{aligned} \therefore \\ f(x) &= (x+1)(x^2 - 9x + 20) \\ &= (x+1)(x-4)(x-5) \end{aligned}$$

- d let  $f(x) = 3x^3 - 4x^2 - 35x + 12$     e let  $f(x) = x^3 + 8$     f let  $f(x) = 12 + 29x + 8x^2 - 4x^3$   
 $f(1) = -24, f(2) = -50,$      $f(1) = 9, f(2) = 16$      $f(1) = 45, f(2) = 70,$   
 $f(-1) = 40, f(-2) = 42$      $f(-1) = 7, f(-2) = 0$      $f(-1) = -5, f(-2) = 18$   
 $f(3) = -48, f(-3) = 0$      $\therefore (x + 2)$  is a factor     $f(3) = 63, f(-3) = 105$   
 $\therefore (x + 3)$  is a factor     $f(4) = 0$   
 $\therefore (x - 4)$  is a factor

$$\begin{array}{r} 3x^2 - 13x + 4 \\ x+3 \overline{) 3x^3 - 4x^2 - 35x + 12} \\ \underline{3x^3 + 9x^2} \phantom{+ 12} \\ -13x^2 - 35x \phantom{+ 12} \\ \underline{-13x^2 - 39x} \phantom{+ 12} \\ 4x + 12 \\ \underline{4x + 12} \\ 0 \end{array}$$

$$\begin{aligned} \therefore \\ f(x) &= (x+3)(3x^2 - 13x + 4) \\ &= (x+3)(3x-1)(x-4) \end{aligned}$$

$$\begin{array}{r} x^2 - 2x + 4 \\ x+2 \overline{) x^3 + 0x^2 + 0x + 8} \\ \underline{x^3 + 2x^2} \phantom{+ 8} \\ -2x^2 + 0x \phantom{+ 8} \\ \underline{-2x^2 - 4x} \phantom{+ 8} \\ 4x + 8 \\ \underline{4x + 8} \\ 0 \end{array}$$

$$\begin{aligned} \therefore \\ f(x) &= (x+2)(x^2 - 2x + 4) \end{aligned}$$

$$\begin{array}{r} -4x^2 - 8x - 3 \\ x-4 \overline{) -4x^3 + 8x^2 + 29x + 12} \\ \underline{-4x^3 + 16x^2} \phantom{+ 12} \\ -8x^2 + 29x \phantom{+ 12} \\ \underline{-8x^2 + 32x} \phantom{+ 12} \\ -3x + 12 \\ \underline{-3x + 12} \\ 0 \end{array}$$

$$\begin{aligned} \therefore \\ f(x) &= (x-4)(-4x^2 - 8x - 3) \\ &= -(x-4)(4x^2 + 8x + 3) \\ &= (4-x)(2x+1)(2x+3) \end{aligned}$$

- 10 a** let  $f(x) = x^3 - x^2 - 10x - 8$   
 $f(1) = -18, f(2) = -24,$   
 $f(-1) = 0$   
 $\therefore (x + 1)$  is a factor
- b** let  $f(x) = x^3 + 2x^2 - 9x - 18$   
 $f(1) = -24, f(2) = -20$   
 $f(-1) = -8, f(-2) = 0$   
 $\therefore (x + 2)$  is a factor
- c** let  $f(x) = 4x^3 - 12x^2 + 9x - 2$   
 $f(1) = -1, f(2) = 0$   
 $\therefore (x - 2)$  is a factor

$$\begin{array}{r}
 x^2 - 2x - 8 \\
 x + 1 \overline{) x^3 - x^2 - 10x - 8} \\
 \underline{x^3 + x^2} \phantom{- 10x - 8} \\
 -2x^2 - 10x \phantom{- 8} \\
 \underline{-2x^2 - 2x} \phantom{- 8} \\
 -8x - 8 \\
 \underline{-8x - 8} \\
 0
 \end{array}$$

$$\begin{aligned}
 &\therefore \\
 &(x + 1)(x^2 - 2x - 8) = 0 \\
 &(x + 1)(x + 2)(x - 4) = 0 \\
 &x = -2, -1, 4
 \end{aligned}$$

$$\begin{array}{r}
 x^2 + 0x - 9 \\
 x + 2 \overline{) x^3 + 2x^2 - 9x - 18} \\
 \underline{x^3 + 2x^2} \phantom{- 9x - 18} \\
 0x^2 - 9x - 18 \\
 \underline{0x^2 + 0x} \phantom{- 18} \\
 -9x - 18 \\
 \underline{-9x - 18} \\
 0
 \end{array}$$

$$\begin{aligned}
 &\therefore \\
 &(x + 2)(x^2 - 9) = 0 \\
 &(x + 2)(x + 3)(x - 3) = 0 \\
 &x = -3, -2, 3
 \end{aligned}$$

$$\begin{array}{r}
 4x^2 - 4x + 1 \\
 x - 2 \overline{) 4x^3 - 12x^2 + 9x - 2} \\
 \underline{4x^3 - 8x^2} \phantom{+ 9x - 2} \\
 -4x^2 + 9x \phantom{- 2} \\
 \underline{-4x^2 + 8x} \phantom{- 2} \\
 x - 2 \\
 \underline{x - 2} \\
 0
 \end{array}$$

$$\begin{aligned}
 &\therefore \\
 &(x - 2)(4x^2 - 4x + 1) = 0 \\
 &(x - 2)(2x - 1)^2 = 0 \\
 &x = \frac{1}{2}, 2
 \end{aligned}$$

- d** let  $f(x) = x^3 - 5x^2 + 3x + 1$   
 $f(1) = 0$   
 $\therefore (x - 1)$  is a factor
- e** let  $f(x) = x^3 + 4x^2 - 9x - 6$   
 $f(1) = -10, f(2) = 0$   
 $\therefore (x - 2)$  is a factor
- f** let  $f(x) = x^3 - 14x + 15$   
 $f(1) = 2, f(2) = -5, f(-1) = 28,$   
 $f(-2) = 35, f(3) = 0$   
 $\therefore (x - 3)$  is a factor

$$\begin{array}{r}
 x^2 - 4x - 1 \\
 x - 1 \overline{) x^3 - 5x^2 + 3x + 1} \\
 \underline{x^3 - x^2} \phantom{+ 3x + 1} \\
 -4x^2 + 3x \phantom{+ 1} \\
 \underline{-4x^2 + 4x} \phantom{+ 1} \\
 -x + 1 \\
 \underline{-x + 1} \\
 0
 \end{array}$$

$$\begin{aligned}
 &\therefore \\
 &(x - 1)(x^2 - 4x - 1) = 0 \\
 &x = 1 \text{ or } \frac{4 \pm \sqrt{16 + 4}}{2} \\
 &x = 1, 2 \pm \sqrt{5}
 \end{aligned}$$

$$\begin{array}{r}
 x^2 + 6x + 3 \\
 x - 2 \overline{) x^3 + 4x^2 - 9x - 6} \\
 \underline{x^3 - 2x^2} \phantom{- 9x - 6} \\
 6x^2 - 9x - 6 \\
 \underline{6x^2 - 12x} \phantom{- 6} \\
 3x - 6 \\
 \underline{3x - 6} \\
 0
 \end{array}$$

$$\begin{aligned}
 &\therefore \\
 &(x - 2)(x^2 + 6x + 3) = 0 \\
 &x = 2 \text{ or } \frac{-6 \pm \sqrt{36 - 12}}{2} \\
 &x = 2, -3 \pm \sqrt{6}
 \end{aligned}$$

$$\begin{array}{r}
 x^2 + 3x - 5 \\
 x - 3 \overline{) x^3 + 0x^2 - 14x + 15} \\
 \underline{x^3 - 3x^2} \phantom{- 14x + 15} \\
 3x^2 - 14x \phantom{+ 15} \\
 \underline{3x^2 - 9x} \phantom{+ 15} \\
 -5x + 15 \\
 \underline{-5x + 15} \\
 0
 \end{array}$$

$$\begin{aligned}
 &\therefore \\
 &(x - 3)(x^2 + 3x - 5) = 0 \\
 &x = 3 \text{ or } \frac{-3 \pm \sqrt{9 + 20}}{2} \\
 &x = 3, \frac{1}{2}(-3 \pm \sqrt{29})
 \end{aligned}$$

- 11 a**  $f(2) = 0$   
 $\therefore 16 - 4 - 30 + c = 0$   
 $c = 18$

**b**

$$\begin{array}{r}
 2x^2 + 3x - 9 \\
 x - 2 \overline{) 2x^3 - x^2 - 15x + 18} \\
 \underline{2x^3 - 4x^2} \phantom{- 15x + 18} \\
 3x^2 - 15x \phantom{+ 18} \\
 \underline{3x^2 - 6x} \phantom{+ 18} \\
 -9x + 18 \\
 \underline{-9x + 18} \\
 0
 \end{array}$$

$$\begin{aligned}
 \therefore f(x) &\equiv (x - 2)(2x^2 + 3x - 9) \\
 &\equiv (x - 2)(2x - 3)(x + 3)
 \end{aligned}$$

- 12 a**  $g(-1) = 0$   
 $\therefore -1 + p + 13 + q = 0$   
 $p + q + 12 = 0 \quad (1)$

$g(3) = 0$   
 $\therefore 27 + 9p - 39 + q = 0$   
 $9p + q - 12 = 0 \quad (2)$   
 $(2) - (1) \Rightarrow 8p - 24 = 0 \Rightarrow p = 3$   
sub (1)  $\Rightarrow 3 + q + 12 = 0 \Rightarrow q = -15$

- b**  $(x + 1)(x - 3)(ax + b) \equiv x^3 + 3x^2 - 13x - 15$   
by inspection  
 $g(x) \equiv (x + 1)(x - 3)(x + 5)$   
 $g(x) = 0 \Rightarrow (x + 1)(x - 3)(x + 5) = 0$   
 $x = -5, -1 \text{ or } 3$

- 13** a  $= f(2) = 8 + 16 - 2 + 6 = 28$   
 c  $= f(-5) = -250 + 25 - 45 + 17 = -163$   
 e  $= f(-\frac{1}{2}) = -\frac{1}{4} - \frac{3}{4} + 10 - 7 = 2$
- 14**  $f(2) = 5$   
 $\therefore 8 - 16 + 10 + c = 5$   
 $c = 3$
- 16** a  $f(-3) = 22$   
 $\therefore -54 + 9a + 13 = 22$   
 $a = 7$   
 b  $f(x) = 2x^3 + 7x^2 + 13$   
 remainder  $= f(4)$   
 $= 128 + 112 + 13$   
 $= 253$
- 18** a  $p(3) = 0$   
 $\therefore 27 + 9a + 27 + b = 0$   
 $9a + b = -54$  (1)  
 b  $p(-2) = -30$   
 $\therefore -8 + 4a - 18 + b = -30$   
 $4a + b = -4$  (2)  
 (1) - (2)  $\Rightarrow 5a = -50$   
 $\therefore a = -10, b = 36$
- 20** a  $g(4) = 39$   
 $\therefore 64 + 4c + 3 = 39$   
 $c = -7$   
 b  $g(x) = x^3 - 7x + 3$
- $$\begin{array}{r}
 x^2 - 2x - 3 \\
 x + 2 \overline{) x^3 + 0x^2 - 7x + 3} \\
 \underline{x^3 + 2x^2} \phantom{+ 3} \\
 -2x^2 - 7x \phantom{+ 3} \\
 \underline{-2x^2 - 4x} \phantom{+ 3} \\
 -3x + 3 \\
 \underline{-3x - 6} \\
 9
 \end{array}$$
- quotient  $= x^2 - 2x - 3$   
 remainder  $= 9$
- 13** b  $= f(-1) = -1 - 2 - 7 + 1 = -9$   
 d  $= f(\frac{1}{2}) = 1 + 1 - 3 - 3 = -4$   
 f  $= f(\frac{2}{3}) = \frac{8}{9} - \frac{8}{3} + \frac{4}{3} - 7 = -7\frac{4}{9}$
- 15**  $f(\frac{1}{2}) = -2$   
 $\therefore \frac{1}{4} - \frac{9}{4} + \frac{1}{2}k + 5 = -2$   
 $k = -10$
- 17** a  $f(-1) = 0$   
 $\therefore -p + q - q + 3 = 0$   
 $p = 3$   
 b  $f(x) = 3x^3 + qx^2 + qx + 3$   
 $f(2) = 15$   
 $\therefore 24 + 4q + 2q + 3 = 15$   
 $q = -2$
- 19**  $f(-1) = 3$   
 $\therefore -4 - 6 - m + n = 3$   
 $n - m = 13$  (1)  
 $f(\frac{1}{2}) = 15$   
 $\therefore \frac{1}{2} - \frac{3}{2} + \frac{1}{2}m + n = 15$   
 $n + \frac{1}{2}m = 16$  (2)  
 (2) - (1)  $\Rightarrow \frac{3}{2}m = 3$   
 $\therefore m = 2, n = 15$