

$$1 \quad \mathbf{a} \quad f(-2) = 0 \Rightarrow -8 - 20 - 2a + b = 0$$

$$\Rightarrow -2a + b = 28 \quad (1)$$

$$f(3) = 0 \Rightarrow 27 - 45 + 3a + b = 0$$

$$\Rightarrow 3a + b = 18 \quad (2)$$

$$(2) - (1) \quad 5a = -10 = 0 \Rightarrow a = -2$$

$$\text{sub. (1)} \quad \Rightarrow b = 24$$

$$\mathbf{b} \quad f(x) \equiv x^3 - 5x^2 - 2x + 24$$

$$(x+2)(x-3)(ax+b) \equiv x^3 - 5x^2 - 2x + 24$$

by inspection

$$f(x) \equiv (x+2)(x-3)(x-4)$$

$$2 \quad f(k) = 8f\left(\frac{1}{2}k\right)$$

$$8k^3 - k^2 + 7 = 8\left(k^3 - \frac{1}{4}k^2 + 7\right)$$

$$8k^3 - k^2 + 7 = 8k^3 - 2k^2 + 56$$

$$k^2 = 49$$

$$k = \pm 7$$

$$3 \quad \mathbf{a} \quad f(2) = 24 - 4 - 24 + 4 = 0$$

$\therefore (x-2)$ is a factor of $f(x)$

b

$$\begin{array}{r} 3x^2 + 5x - 2 \\ x-2 \overline{) 3x^3 - x^2 - 12x + 4} \\ \underline{3x^3 - 6x^2} \\ 5x^2 - 12x \\ \underline{5x^2 - 10x} \\ -2x + 4 \\ \underline{-2x + 4} \\ 0 \end{array}$$

$$\therefore f(x) = (x-2)(3x^2 + 5x - 2)$$

$$= (x-2)(3x-1)(x+2)$$

$$f(x) = 0 \Rightarrow (x-2)(3x-1)(x+2) = 0$$

$$x = -2, \frac{1}{3} \text{ or } 2$$

$$4 \quad 6 + 7x - x^3 = 0$$

$$\text{let } f(x) = 6 + 7x - x^3$$

$$f(1) = 12, f(2) = 12, f(-1) = 0$$

$\therefore (x+1)$ is a factor of $f(x)$

$$\begin{array}{r} -x^2 + x + 6 \\ x+1 \overline{) -x^3 + 0x^2 + 7x + 6} \\ \underline{-x^3 - x^2} \\ x^2 + 7x \\ \underline{x^2 + x} \\ 6x + 6 \\ \underline{6x + 6} \\ 0 \end{array}$$

$$\therefore (x+1)(-x^2 + x + 6) = 0$$

$$-(x+1)(x-3)(x+2) = 0$$

$$x = -2, -1, 3$$

$$\therefore (-2, 0), (-1, 0) \text{ and } (3, 0)$$

5 a $f(-1) = -4$

$$\therefore -3 + p - 8 + q = -4$$

$$p + q = 7 \quad (1)$$

$$f(2) = 80$$

$$\therefore 24 + 4p + 16 + q = 80$$

$$4p + q = 40 \quad (2)$$

$$(2) - (1) \Rightarrow 3p = 33$$

$$\therefore p = 11, q = -4$$

b $f(x) \equiv 3x^3 + 11x^2 + 8x - 4$

$$f(-2) = -24 + 44 - 16 - 4 = 0$$

$$\therefore (x + 2) \text{ is a factor}$$

c

$$\begin{array}{r} x+2 \overline{) 3x^2 + 5x - 2} \\ \underline{3x^3 + 11x^2 + 8x - 4} \\ 3x^3 + 6x^2 \\ \underline{ + 5x^2 + 8x} \\ 5x^2 + 10x \\ \underline{ - 2x - 4} \\ -2x - 4 \\ \underline{ - 2x - 4} \end{array}$$

$$\therefore f(x) = (x + 2)(3x^2 + 5x - 2)$$

$$= (3x - 1)(x + 2)^2$$

$$\therefore f(x) = 0 \Rightarrow x = -2 \text{ or } \frac{1}{3}$$

7 a $= f(-1) = -1 + 7 - 14 + 3 = -5$

b

$$\begin{array}{r} n+1 \overline{) n^2 + 6n + 8} \\ \underline{n^3 + 7n^2 + 14n + 3} \\ n^3 + n^2 \\ \underline{ + 6n^2 + 14n} \\ 6n^2 + 6n \\ \underline{ + 8n + 3} \\ 8n + 8 \\ \underline{ - 5} \end{array}$$

$$\therefore f(n) = (n + 1)(n^2 + 6n + 8) - 5$$

$$f(n) = (n + 1)(n + 2)(n + 4) - 5$$

c $(n + 1)$ and $(n + 2)$ are consecutive integers

$$\therefore \text{either } (n + 1) \text{ or } (n + 2) \text{ is even}$$

$$\therefore (n + 1)(n + 2)(n + 4) \text{ is even}$$

$$\therefore (n + 1)(n + 2)(n + 4) - 5 \text{ is odd}$$

6 a let $f(x) = x^3 - 4x^2 - 7x + 10$

$$f(1) = 1 - 4 - 7 + 10 = 0$$

$$\therefore (x - 1) \text{ is a factor}$$

$$\begin{array}{r} x-1 \overline{) x^2 - 3x - 10} \\ \underline{x^3 - 4x^2 - 7x + 10} \\ x^3 - x^2 \\ \underline{ - 3x^2 - 7x} \\ -3x^2 + 3x \\ \underline{ - 10x + 10} \\ -10x + 10 \\ \underline{ - 10x + 10} \end{array}$$

$$\therefore (x - 1)(x^2 - 3x - 10) = 0$$

$$(x - 1)(x + 2)(x - 5) = 0$$

$$x = -2, 1, 5$$

b $y^2 = x$ in part a

$$y^2 = 1, 5 \text{ or } -2 \text{ [no solutions]}$$

$$y = \pm 1, \pm \sqrt{5}$$