

- 1 **a** $x^2 + y^2 = 25$ **b** $(x-1)^2 + (y-3)^2 = 4$ **c** $(x-4)^2 + (y+6)^2 = 1$
 d $(x+1)^2 + (y+8)^2 = 9$ **e** $(x+\frac{1}{2})^2 + (y-\frac{1}{2})^2 = \frac{1}{4}$ **f** $(x+3)^2 + (y-9)^2 = 12$
- 2 **a** centre (0, 0) radius 4 **b** centre (6, 1) radius 9 **c** centre (-1, 4) radius 11
 d centre (7, 0) radius 0.3 **e** centre (-2, -5) radius $4\sqrt{2}$ **f** centre (8, -9) radius $6\sqrt{3}$
- 3 **a** $x^2 + (y-2)^2 - 4 + 3 = 0$ **b** $(x-1)^2 - 1 + (y-5)^2 - 25 - 23 = 0$
 $x^2 + (y-2)^2 = 1$ $(x-1)^2 + (y-5)^2 = 49$
 centre (0, 2) radius 1 centre (1, 5) radius 7
- c** $(x+6)^2 - 36 + (y-4)^2 - 16 + 36 = 0$ **d** $(x-1)^2 - 1 + (y+8)^2 - 64 = 35$
 $(x+6)^2 + (y-4)^2 = 16$ $(x-1)^2 + (y+8)^2 = 100$
 centre (-6, 4) radius 4 centre (1, -8) radius 10
- e** $(x-4)^2 - 16 + (y+3)^2 - 9 = 0$ **f** $(x+5)^2 - 25 + (y-1)^2 - 1 - 19 = 0$
 $(x-4)^2 + (y+3)^2 = 25$ $(x+5)^2 + (y-1)^2 = 45$
 centre (4, -3) radius 5 centre (-5, 1) radius $3\sqrt{5}$
- g** $x^2 + y^2 - x - 6y + \frac{1}{4} = 0$ **h** $x^2 + y^2 + \frac{2}{3}x - \frac{8}{3}y + \frac{8}{9} = 0$
 $(x-\frac{1}{2})^2 - \frac{1}{4} + (y-3)^2 - 9 + \frac{1}{4} = 0$ $(x+\frac{1}{3})^2 - \frac{1}{9} + (y-\frac{4}{3})^2 - \frac{16}{9} + \frac{8}{9} = 0$
 $(x-\frac{1}{2})^2 + (y-3)^2 = 9$ $(x+\frac{1}{3})^2 + (y-\frac{4}{3})^2 = 1$
 centre $(\frac{1}{2}, 3)$ radius 3 centre $(-\frac{1}{3}, \frac{4}{3})$ radius 1
- 4 **a** radius = $\sqrt{9+16} = 5$ $\therefore (x-1)^2 + (y+2)^2 = 25$
 b radius = $\sqrt{25+4} = \sqrt{29}$ $\therefore (x+5)^2 + (y-7)^2 = 29$
- 5 **a** centre $(\frac{1+3}{2}, -2) = (2, -2)$ **b** centre $(\frac{-7+1}{2}, \frac{2+8}{2}) = (-3, 5)$ **c** centre $(\frac{1+4}{2}, \frac{1+0}{2}) = (\frac{5}{2}, \frac{1}{2})$
 radius = 1 radius = $\sqrt{16+9} = 5$ radius = $\sqrt{\frac{9}{4} + \frac{1}{4}} = \sqrt{\frac{5}{2}}$
 $\therefore (x-2)^2 + (y+2)^2 = 1$ $\therefore (x+3)^2 + (y-5)^2 = 25$ $\therefore (x-\frac{5}{2})^2 + (y-\frac{1}{2})^2 = \frac{5}{2}$
- 6 **a** grad $PQ = \frac{10-1}{3-0} = 3$, grad $QR = \frac{9-10}{6-3} = -\frac{1}{3}$
 grad $PQ \times$ grad $QR = 3 \times (-\frac{1}{3}) = -1$
 $\therefore PQ$ and QR are perpendicular
 $\therefore \angle PQR$ is a right-angle
- b** $\angle PQR$ is a right-angle $\therefore PR$ is a diameter of C
 \therefore centre is $(\frac{0+6}{2}, \frac{1+9}{2}) = (3, 5)$
 radius = 5
 $\therefore (x-3)^2 + (y-5)^2 = 25$
 $x^2 - 6x + 9 + y^2 - 10y + 25 - 25 = 0$
 $x^2 + y^2 - 6x - 10y + 9 = 0$

- 7 a** centre (0, 0) radius 8
dist. pt to centre = 9
 \therefore outside circle
- c** $(x+5)^2 - 25 + (y-2)^2 - 4 = 140$
 $(x+5)^2 + (y-2)^2 = 169$
centre (-5, 2) radius 13
dist. pt to centre = $\sqrt{144+25} = 13$
 \therefore on circle
- 8** $(x+6)^2 - 36 + (y-3)^2 - 9 + 27 = 0$
 $(x+6)^2 + (y-3)^2 = 18$
centre (-6, 3) radius $3\sqrt{2}$
dist. Q to centre = $\sqrt{196+4} = 10\sqrt{2}$
min. $PQ = 10\sqrt{2} - 3\sqrt{2} = 7\sqrt{2}$
- 10** $(x+4)^2 - 16 + (y-6)^2 - 36 + k = 0$
 $(x+4)^2 + (y-6)^2 = 52 - k$
centre (-4, 6) $r^2 = 52 - k$
 $r > 0 \therefore k < 52$
also require $r < 4$
 $\therefore 52 - k < 16$
 $k > 36$
 $\therefore 36 < k < 52$
- 12 a** $(x-2)^2 - 4 + (y-2)^2 - 4 - 28 = 0$
 $(x-2)^2 + (y-2)^2 = 36$
centre (2, 2) radius 6
dist. = $\sqrt{64+36} = 10$
- b** tangent perp to radius
 $\therefore AB^2 = 10^2 - 6^2 = 64$
 $AB = 8$
- b** $(x-1)^2 - 1 + (y-3)^2 - 9 - 26 = 0$
 $(x-1)^2 + (y-3)^2 = 36$
centre (1, 3) radius 6
dist. pt to centre = $\sqrt{9+16} = 5$
 \therefore inside circle
- d** $(x+1)^2 - 1 + (y+4)^2 - 16 - 13 = 0$
 $(x+1)^2 + (y+4)^2 = 30$
centre (-1, -4) radius $\sqrt{30}$
dist. pt to centre = $\sqrt{9+25} = \sqrt{34}$
 \therefore outside circle
- 9** x -coord of centre = $\frac{2+8}{2} = 5$
 y -coord of centre = 4 \therefore centre (5, 4)
radius = dist. (0, 4) to (5, 4) = 5
 $\therefore (x-5)^2 + (y-4)^2 = 25$
- 11 a** mid-point $PQ = (\frac{-2+2}{2}, \frac{-2+(-4)}{2}) = (0, -3)$
grad $PQ = \frac{-4+2}{2+2} = -\frac{1}{2}$
perp. grad = 2
 $\therefore y = 2x - 3$
- b** mid-point $PR = (\frac{-2+7}{2}, \frac{-2+1}{2}) = (\frac{5}{2}, -\frac{1}{2})$
grad $PR = \frac{1+2}{7+2} = \frac{1}{3}$
perp. grad = -3
perp. bisector $y + \frac{1}{2} = -3(x - \frac{5}{2})$
 $y = 7 - 3x$
centre where intersect $2x - 3 = 7 - 3x$
 $x = 2 \therefore (2, 1)$
- c** radius = dist. (2, 1) to (7, 1) = 5
 $\therefore (x-2)^2 + (y-1)^2 = 25$
- 13** $(x+3)^2 - 9 + (y-1)^2 - 1 = 0$
 $(x+3)^2 + (y-1)^2 = 10$
centre (-3, 1) radius $\sqrt{10}$
dist. centre to (2, 6) = $\sqrt{25+25} = \sqrt{50}$
 $PQ^2 = (\sqrt{50})^2 - (\sqrt{10})^2 = 40$
 $PQ = \sqrt{40} = 2\sqrt{10}$

14 a $(x-3)^2 - 9 + (y-5)^2 - 25 + 16 = 0$
 \therefore centre (3, 5)

b $\text{grad} = \frac{5-2}{3-6} = -1$

c $y-2 = -(x-6)$ [$y = 8 - x$]

15 a $(x+2)^2 - 4 + y^2 = 13$
 \therefore centre (-2, 0)

$\text{grad} = \frac{0-4}{-2+1} = 4$

$\therefore y-4 = 4(x+1)$ [$y = 4x + 8$]

b $(x+1)^2 - 1 + (y+2)^2 - 4 - 40 = 0$
 \therefore centre (-1, -2)

$\text{grad normal} = \frac{-2-1}{-1-5} = \frac{1}{2}$

\therefore grad tangent = -2

$\therefore y-1 = -2(x-5)$ [$y = 11 - 2x$]

c $(x-5)^2 - 25 + (y+2)^2 - 4 + 4 = 0$
 \therefore centre (5, -2)

$\text{grad normal} = \frac{-2-2}{5-2} = -\frac{4}{3}$

\therefore grad tangent = $\frac{3}{4}$

$\therefore y-2 = \frac{3}{4}(x-2)$ [$3x - 4y + 2 = 0$]

16 $x=0 \Rightarrow y^2 + 6y - 16 = 0$
 $(y+8)(y-2) = 0$
 $y = -8, 2$

$y=0 \Rightarrow x^2 - 6x - 16 = 0$
 $(x+2)(x-8) = 0$
 $x = -2, 8$

\therefore (0, -8), (0, 2), (-2, 0) and (8, 0)

17 a sub. $x^2 + (x-4)^2 = 10$
 $x^2 - 4x + 3 = 0$
 $(x-1)(x-3) = 0$
 $x = 1, 3$

\therefore (1, -3) and (3, -1)

b sub. $y = 17 - 3x$
 $x^2 + (17 - 3x)^2 - 4x - 2(17 - 3x) - 15 = 0$
 $x^2 - 10x + 24 = 0$
 $(x-4)(x-6) = 0$
 $x = 4, 6$

\therefore (4, 5) and (6, -1)

c sub.
 $4x^2 + 4(2x+2)^2 + 4x - 8(2x+2) - 15 = 0$
 $4x^2 + 4x - 3 = 0$
 $(2x+3)(2x-1) = 0$
 $x = -\frac{3}{2}, \frac{1}{2}$

\therefore $(-\frac{3}{2}, -1)$ and $(\frac{1}{2}, 3)$

18 sub.
 $x^2 + (1-x)^2 + 6x + 2(1-x) = 27$
 $x^2 + x - 12 = 0$
 $(x+4)(x-3) = 0$
 $x = -4, 3$
 \therefore (-4, 5) and (3, -2)
 $AB = \sqrt{49+49} = 7\sqrt{2}$

19 sub.
 $x^2 + (2x+1)^2 - 8x - 8(2x+1) + 27 = 0$
 $x^2 - 4x + 4 = 0$
 $(x-2)^2 = 0$
 repeated root \therefore tangent
 touch when $x = 2$ \therefore at (2, 5)

20 sub.

$$x^2 + (x+k)^2 + 6x - 8(x+k) + 17 = 0$$

$$2x^2 + (2k-2)x + k^2 - 8k + 17 = 0$$

tangent \therefore repeated root $\therefore b^2 - 4ac = 0$

$$\Rightarrow (2k-2)^2 - 8(k^2 - 8k + 17) = 0$$

$$k^2 - 14k + 33 = 0$$

$$(k-3)(k-11) = 0$$

$$\therefore k = 3 \text{ or } 11$$

22 sub. $x = \frac{k-3y}{2}$

$$\left(\frac{k-3y}{2}\right)^2 + y^2 + 6\left(\frac{k-3y}{2}\right) + 4y = 0$$

$$(k-3y)^2 + 4y^2 + 12(k-3y) + 16y = 0$$

$$13y^2 - (6k+20)y + k^2 + 12k = 0$$

tangent \therefore repeated root $\therefore b^2 - 4ac = 0$

$$\Rightarrow (6k+20)^2 - 52(k^2 + 12k) = 0$$

$$k^2 + 24k - 25 = 0$$

$$(k+25)(k-1) = 0$$

$$\therefore k = -25, 1$$

21 sub.

$$x^2 + m^2x^2 - 8x - 16mx + 72 = 0$$

$$(1+m^2)x^2 - (8+16m)x + 72 = 0$$

tangent \therefore repeated root $\therefore b^2 - 4ac = 0$

$$\Rightarrow (8+16m)^2 - 288(1+m^2) = 0$$

$$m^2 - 8m + 7 = 0$$

$$(m-1)(m-7) = 0$$

$$\therefore m = 1, 7$$

23 a $x = 0 \Rightarrow y^2 - 6y - 7 = 0$

$$(y+1)(y-7) = 0$$

$$y = -1, 7$$

 $\therefore (0, -1)$ and $(0, 7)$ b $(x-2)^2 - 4 + (y-3)^2 - 9 = 7$ \therefore centre $(2, 3)$

$$\text{grad normal at } (0, -1) = \frac{3+1}{2-0} = 2$$

 \therefore grad tangent at $(0, -1) = -\frac{1}{2}$

$$\therefore y = -\frac{1}{2}x - 1$$

$$\text{grad normal at } (0, 7) = \frac{3-7}{2-0} = -2$$

 \therefore grad tangent at $(0, 7) = \frac{1}{2}$

$$\therefore y = \frac{1}{2}x + 7$$

$$\text{intersect when } -\frac{1}{2}x - 1 = \frac{1}{2}x + 7$$

$$x = -8$$

 $\therefore (-8, 3)$