

- 1 a $(x-3)^2 + (y+2)^2 = 25$
 b sub. $(x-3)^2 + [(2x-3)+2]^2 = 25$
 $(x-3)^2 + (2x-1)^2 = 25$
 $x^2 - 2x - 3 = 0$
 $(x+1)(x-3) = 0$
 $x = -1, 3$
 $\therefore (-1, -5)$ and $(3, 3)$
 $AB^2 = 4^2 + 8^2 = 80$
 $AB = \sqrt{80} = \sqrt{16 \times 5} = 4\sqrt{5}$
- 2 a $= \left(\frac{-5+3}{2}, \frac{6+8}{2}\right) = (-1, 7)$
 b radius $= \sqrt{16+1} = \sqrt{17}$
 $\therefore (x+1)^2 + (y-7)^2 = 17$
 c grad of radius $= \frac{7-6}{-1-(-5)} = \frac{1}{4}$
 \therefore grad of tangent $= -4$
 $\therefore y-6 = -4(x+5)$
 $[y = -4x - 14]$
- 3 a $(x+4)^2 - 16 + (y-8)^2 - 64 + 62 = 0$
 $(x+4)^2 + (y-8)^2 = 18$
 \therefore centre $(-4, 8)$ radius $3\sqrt{2}$
 b grad of $l = 2 \therefore$ grad of perp. $= -\frac{1}{2}$
 eqn. of line perp to l through centre:
 $y-8 = -\frac{1}{2}(x+4)$
 $y = 6 - \frac{1}{2}x$
 intersects l when:
 $2x+1 = 6 - \frac{1}{2}x$
 $x = 2 \therefore (2, 5)$ is closest point
 dist. $(2, 5)$ to centre
 $= \sqrt{36+9} = \sqrt{45} = 3\sqrt{5}$
 min. dist. $= 3\sqrt{5} - 3\sqrt{2} = 3(\sqrt{5} - \sqrt{2})$
- 4 a $PQ = \sqrt{1+9} = \sqrt{10}$
 radius $= \frac{1}{2}PQ = \frac{1}{2}\sqrt{10}$
 b = midpoint of PR
 $= \left(\frac{0+7}{2}, \frac{4+3}{2}\right) = \left(\frac{7}{2}, \frac{7}{2}\right)$
 c midpoint of $PQ = \left(\frac{0+1}{2}, \frac{4+1}{2}\right) = \left(\frac{1}{2}, \frac{5}{2}\right)$
 centre of $C_1 =$ midpoint of $\left(\frac{1}{2}, \frac{5}{2}\right)$ and $\left(\frac{7}{2}, \frac{7}{2}\right)$
 $= \left(\frac{\frac{1}{2}+\frac{7}{2}}{2}, \frac{\frac{5}{2}+\frac{7}{2}}{2}\right) = (2, 3)$
 \therefore eqn. of C_1 :
 $(x-2)^2 + (y-3)^2 = \left(\frac{1}{2}\sqrt{10}\right)^2$
 $x^2 - 4x + 4 + y^2 - 6y + 9 = \frac{5}{2}$
 $2x^2 - 8x + 8 + 2y^2 - 12y + 18 = 5$
 $2x^2 + 2y^2 - 8x - 12y + 21 = 0$
- 5 a midpoint $AB = \left(\frac{0+2}{2}, \frac{3+7}{2}\right) = (1, 5)$
 grad $AB = \frac{7-3}{2-0} = 2$
 \therefore perp. grad $= -\frac{1}{2}$
 $\therefore y-5 = -\frac{1}{2}(x-1)$
 $[y = \frac{11}{2} - \frac{1}{2}x]$
 b circle touches y -axis at $(0, 3)$
 \therefore y -coord of centre $= 3$
 sub. $3 = \frac{11}{2} - \frac{1}{2}x$
 $x = 5$
 \therefore centre $(5, 3)$ radius 5
 $\therefore (x-5)^2 + (y-3)^2 = 25$
 c grad of radius $= \frac{7-3}{2-5} = -\frac{4}{3}$
 \therefore grad of tangent $= \frac{3}{4}$
 $\therefore y-7 = \frac{3}{4}(x-2)$
 $4y-28 = 3x-6$
 $3x-4y+22 = 0$
- 6 $AP^2 = (x+3)^2 + (y-4)^2$
 $BP^2 = x^2 + (y+2)^2$
 $AP = 2BP \therefore AP^2 = 4BP^2$
 $\therefore (x+3)^2 + (y-4)^2 = 4[x^2 + (y+2)^2]$
 $x^2 + 6x + 9 + y^2 - 8y + 16 = 4x^2 + 4y^2 + 16y + 16$
 $x^2 - 2x + y^2 + 8y - 3 = 0$
 $(x-1)^2 - 1 + (y+4)^2 - 16 - 3 = 0$
 $(x-1)^2 + (y+4)^2 = 20$
 in form $(x-a)^2 + (y-b)^2 = r^2 \therefore$ circle
 centre $(1, -4)$ radius $2\sqrt{5}$

$$7 \quad \mathbf{a} \quad = \left(\frac{-4+(-2)}{2}, \frac{9+(-5)}{2} \right) = (-3, 2)$$

$$\mathbf{b} \quad \text{radius} = \sqrt{1+49} = \sqrt{50}$$

$$\therefore (x+3)^2 + (y-2)^2 = 50$$

\mathbf{c} sub. (2, 7) into eqn of C:

$$(2+3)^2 + (7-2)^2 = 50$$

$$25 + 25 = 50$$

true $\therefore R$ lies on C

\mathbf{d} 90°

PQ is a diameter

$\therefore \angle PRQ$ is the angle in a semicircle

$$8 \quad \mathbf{a} \quad x^2 + (y-2)^2 - 4 - 16 = 0$$

\therefore centre (0, 2)

$$\mathbf{b} \quad C_2: (x-1)^2 - 1 + (y-4)^2 - 16 - 60 = 0$$

\therefore centre (1, 4)

$$\text{grad} = \frac{4-2}{1-0} = 2$$

$\therefore y = 2x + 2$

\mathbf{c} sub. into eqn of C_1 :

$$x^2 + [(2x+2)-2]^2 - 20 = 0$$

$$x^2 + (2x)^2 - 20 = 0$$

$$x^2 = 4$$

$$x = \pm 2$$

from diagram, $x = -2$ at P

$\therefore P(-2, -2)$

l perp to line through centres

\therefore grad = $-\frac{1}{2}$

$$\therefore y + 2 = -\frac{1}{2}(x + 2)$$

$$[y = -\frac{1}{2}x - 3]$$

$$9 \quad \mathbf{a} \quad (x-4)^2 - 16 + (y+2)^2 - 4 + 12 = 0$$

$$(x-4)^2 + (y+2)^2 = 8$$

centre (4, -2) radius $2\sqrt{2}$

\mathbf{b} dist. P to centre

$$= \sqrt{1+49} = \sqrt{50} = 5\sqrt{2}$$

$$\therefore \text{max. } PQ = 5\sqrt{2} + 2\sqrt{2} = 7\sqrt{2}$$

$$\text{min. } PQ = 5\sqrt{2} - 2\sqrt{2} = 3\sqrt{2}$$

\mathbf{c} tangent perp. to radius

$$PQ^2 = (5\sqrt{2})^2 - (2\sqrt{2})^2 = 50 - 8 = 42$$

$$PQ = \sqrt{42} = 6.48$$

$$10 \quad \mathbf{a} \quad \text{radius} = b$$

$$\therefore (x-a)^2 + (y-b)^2 = b^2$$

\mathbf{b} sub. $y = x$ into eqn

$$(x-a)^2 + (x-b)^2 = b^2$$

$$x^2 - 2ax + a^2 + x^2 - 2bx + b^2 = b^2$$

$$2x^2 - 2(a+b)x + a^2 = 0$$

tangent \therefore repeated root

$$\therefore "b^2 - 4ac" = 0$$

$$4(a+b)^2 - 8a^2 = 0$$

$$a^2 - 2ab - b^2 = 0$$

$$a = \frac{2b \pm \sqrt{4b^2 + 4b^2}}{2} = b \pm \sqrt{2}b$$

$$a > 0, b > 0 \quad \therefore a = (1 + \sqrt{2})b$$