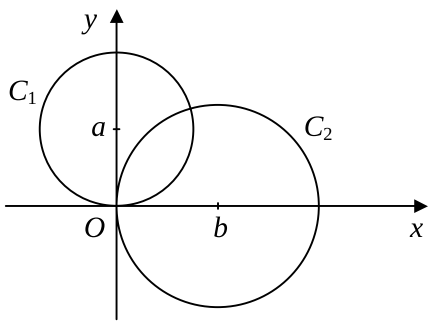
1 **a** $(x-4)^2 - 16 + y^2 + 7 = 0$
$\therefore$ centre (4, 0)
b $(x-4)^2 + y^2 = 9$
$\therefore$ radius = 3

2 **a** $(x-3)^2 - 9 + (y+1)^2 - 1 - 15 = 0$
$\therefore$ centre (3, –1)
b $(x-3)^2 + (y+1)^2 = 25$
$\therefore$ radius = 5
c grad of radius = $\frac{2-(-1)}{7-3} = \frac{3}{4}$
$\therefore$ grad of tangent = $-\frac{4}{3}$
$\therefore$ $y - 2 = -\frac{4}{3}(x-7)$
$3y - 6 = -4x + 28$
$4x + 3y - 34 = 0$

3 **a** $(x+3)^2 - 9 + (y-4)^2 - 16 + 21 = 0$
$(x+3)^2 + (y-4)^2 = 4$
$\therefore$ centre (–3, 4) radius 2
b dist. of centre from O = $\sqrt{9+16} = 5$
$\therefore$ max. dist. of P from O
= 5 + 2 = 7

4 **a** centre (0, 0) $\therefore$ grad of radius = 1
$\therefore$ grad of tangent = –1
$\therefore$ $y - 5 = -(x-5)$ [$y = 10 - x$]
b grad of radius = –7
$\therefore$ grad of tangent = $\frac{1}{7}$
$\therefore$ $y + 7 = \frac{1}{7}(x-1)$
$7y + 49 = x - 1$
$x - 7y - 50 = 0$
c sub. $x - 7(10 - x) - 50 = 0$
$x = 15$
$\therefore$ (15, –5)

5 **a** $x^2 + (y-a)^2 - a^2 = 0$
$x^2 + (y-a)^2 = a^2$
$\therefore$ centre $(0, a)$ radius a
b C_2: $(x-b)^2 - b^2 + y^2 = 0$
$(x-b)^2 + y^2 = b^2$, centre $(b, 0)$ radius b



6 **a** $(x+1)^2 - 1 + (y-7)^2 - 49 + 30 = 0$
$\therefore$ centre (–1, 7)
b $(x+1)^2 + (y-7)^2 = 20$
$\therefore$ radius = $\sqrt{20} = 2\sqrt{5}$
c sub. $y = 2x - 1$ into eqn. of circle
$x^2 + (2x-1)^2 + 2x - 14(2x-1) + 30 = 0$
$x^2 - 6x + 9 = 0$
$(x-3)^2 = 0$
repeated root $\therefore$ tangent
point of contact (3, 5)

7 **a** $(x-3)^2 - 9 + (y-6)^2 - 36 + 28 = 0$
$\therefore$ centre (3, 6)
b sub.
$x^2 + (x-2)^2 - 6x - 12(x-2) + 28 = 0$
$x^2 - 11x + 28 = 0$
$(x-4)(x-7) = 0$
$x = 4, 7$
$\therefore$ A (4, 2), B (7, 5)
$\therefore$ $AB = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

8 **a** radius = $\sqrt{16+4} = \sqrt{20}$
$\therefore$ $(x-8)^2 + (y+1)^2 = 20$
b sub. $x = -2y - 4$ into eqn. of circle:
$(-2y-12)^2 + (y+1)^2 = 20$
$4y^2 + 48y + 144 + y^2 + 2y + 1 = 20$
$y^2 + 10y + 25 = 0$
$(y+5)^2 = 0$
repeated root $\therefore$ tangent

9 **a** grad $PQ = \frac{14-2}{8+10} = \frac{2}{3}$

grad $PR = \frac{-10-2}{-2+10} = -\frac{3}{2}$

grad $PR \times$ grad $PQ = -\frac{3}{2} \times \frac{2}{3} = -1$

$\therefore$ PR is perpendicular to PQ

b $\angle QPR = 90°$ $\therefore$ QR is a diameter of the circle

$\therefore$ centre of circle is mid-point of QR

$= (\frac{8-2}{2}, \frac{14-10}{2}) = (3, 2)$

radius $= \sqrt{25+144} = 13$

$\therefore$ $(x-3)^2 + (y-2)^2 = 169$

$x^2 - 6x + 9 + y^2 - 4y + 4 - 169 = 0$

$x^2 + y^2 - 6x - 4y - 156 = 0$

10 **a** $(x-1)^2 - 1 + (y - \frac{7}{2})^2 - \frac{49}{4} - 16 = 0$

$\therefore$ centre $(1, \frac{7}{2})$

b $(x-1)^2 + (y - \frac{7}{2})^2 = \frac{117}{4}$

$\therefore$ radius $= \sqrt{\frac{117}{4}} = \sqrt{\frac{9\times13}{4}} = \frac{3}{2}\sqrt{13}$ $[k = \frac{3}{2}]$

c grad of radius $= \frac{8-\frac{7}{2}}{4-1} = \frac{3}{2}$

$\therefore$ grad of tangent $= -\frac{2}{3}$

$\therefore$ $y - 8 = -\frac{2}{3}(x-4)$

$3y - 24 = -2x + 8$

$2x + 3y - 32 = 0$

11 **a** grad of $x - 2y + 3 = 0$ is $\frac{1}{2}$

$\therefore$ grad of perp bisector $= -2$

passes through centre of circle

$\therefore$ $y - 7 = -2(x-6)$

$y = -2x + 19$

mid-point of chord where intersect

$x - 2(-2x + 19) + 3 = 0$

$x = 7$ $\therefore$ $(7, 5)$

b $3 - 2y + 3 = 0$

$\therefore$ $y = 3$ $\therefore$ A $(3, 3)$

let B be (p, q)

$\therefore$ $(\frac{3+p}{2}, \frac{3+q}{2}) = (7, 5)$

$p = 11, q = 7$ $\therefore$ B $(11, 7)$

c radius $= \sqrt{9+16} = 5$

$\therefore$ $(x-6)^2 + (y-7)^2 = 25$

12 **a** $(x-4)^2 - 16 + (y-8)^2 - 64 + 72 = 0$

$(x-4)^2 + (y-8)^2 = 8$

$\therefore$ centre $(4, 8)$ radius $2\sqrt{2}$

b $= \sqrt{16+64} = \sqrt{80} = 4\sqrt{5}$

c tangent perp. to radius

$\therefore$ $OA^2 = (\sqrt{80})^2 - (2\sqrt{2})^2 = 72$

$OA = \sqrt{72} = \sqrt{36\times2} = 6\sqrt{2}$

13 **a** C: $(x-2)^2 - 4 + y^2 - 6 = 0$

$\therefore$ centre $(2, 0)$

l: when $x = 2$, $y = 3(2) - 6 = 0$

$\therefore$ l passes through centre of C

b eqn. of tangent: $y = 3x + k$

sub. into eqn. of circle:

$x^2 + (3x + k)^2 - 4x - 6 = 0$

$10x^2 + (6k - 4)x + k^2 - 6 = 0$

tangent $\therefore$ repeated root $\therefore$ $b^2 - 4ac = 0$

$(6k-4)^2 - 40(k^2 - 6) = 0$

$k^2 + 12k - 64 = 0$

$(k + 16)(k - 4) = 0$

$k = -16, 4$

$\therefore$ $y = 3x - 16$ and $y = 3x + 4$