1 **a** volume $= 2x^2h = 4000$

$\therefore\ h = \frac{2000}{x^2}$

b $A = 2x^2 + 2(2xh) + 2(xh)$

$= 2x^2 + 6xh$

$= 2x^2 + (6x \times \frac{2000}{x^2})$

$= 2x^2 + \frac{12000}{x}$

c $\frac{dA}{dx} = 4x - 12\,000x^{-2}$

SP: $4x - 12\,000x^{-2} = 0$

$x^3 = 3000$

$x = \sqrt[3]{3000} = 14.4$ (3sf)

d min $A = 1250$ (3sf)

e $\frac{d^2A}{dx^2} = 4 + 24\,000x^{-3}$

when $x = \sqrt[3]{3000}$, $\frac{d^2A}{dx^2} = 12$

$\frac{d^2A}{dx^2} > 0$ $\therefore$ minimum

2 **a** S.A. $= 2\pi r^2 + 2\pi rh = 30\,000$

$\therefore\ \pi rh = 15\,000 - \pi r^2$

$h = \frac{15000}{\pi r} - r$

$V = \pi r^2 h$

$= \pi r^2(\frac{15000}{\pi r} - r)$

$= 15\,000r - \pi r^3$

b $\frac{dV}{dr} = 15\,000 - 3\pi r^2$

SP: $15\,000 - 3\pi r^2 = 0$

$r^2 = \frac{5000}{\pi}$

$r = \sqrt{\frac{5000}{\pi}}$ [= 39.9 (3sf)]

max volume = 399 000 cm^3 (3sf)

$\frac{d^2V}{dr^2} = -6\pi r$

when $r = \sqrt{\frac{5000}{\pi}}$, $\frac{d^2V}{dr^2} = -752$

$\frac{d^2V}{dr^2} < 0$ $\therefore$ maximum

3 **a** S.A. $= 2x^2 + 4xl = k$

$\therefore\ 4xl = k - 2x^2$

$l = \frac{k - 2x^2}{4x}$

b $V = x^2 l$

$= x^2 \times \frac{k - 2x^2}{4x}$

$= \frac{1}{4}kx - \frac{1}{2}x^3$

$\frac{dV}{dx} = \frac{1}{4}k - \frac{3}{2}x^2$

SP: $\frac{1}{4}k - \frac{3}{2}x^2 = 0$

$x^2 = \frac{1}{6}k$

$x = \sqrt{\frac{k}{6}}$

$\frac{d^2V}{dx^2} = -3x$

when $x = \sqrt{\frac{k}{6}}$, $\frac{d^2V}{dx^2} < 0$ $\therefore$ maximum

$l = \frac{k - \frac{1}{3}k}{4\sqrt{\frac{k}{6}}} = \frac{2}{3}k \times \frac{1}{4} \times \sqrt{\frac{6}{k}}$

$= \frac{k}{6} \times \sqrt{\frac{6}{k}} = \sqrt{\frac{k}{6}}$

$\therefore$ maximum V when $l = x$ $\therefore$ prism is a cube