

1 a $f'(x) = 24 + 6x - 3x^2$

b $24 + 6x - 3x^2 \geq 0$

$$x^2 - 2x - 8 \leq 0$$

$$(x + 2)(x - 4) \leq 0$$

$$-2 \leq x \leq 4$$

2 a $(-2, 30) \Rightarrow 30 = -8 + 4a + 48 + b$

$$\therefore 4a + b + 10 = 0$$

b $\frac{dy}{dx} = 3x^2 + 2ax - 24$

SP at P $\therefore \frac{dy}{dx} = 0$

$$\Rightarrow 12 - 4a - 24 = 0$$

$$a = -3, b = 2$$

c $3x^2 - 6x - 24 = 0$

$$3(x + 2)(x - 4) = 0$$

$$x = -2 \text{ (at P) or } 4$$

$$\text{other SP } (4, -78)$$

3 a $f'(x) = 2x - 16x^{-2}$

b SP: $2x - 16x^{-2} = 0$

$$x^3 = 8$$

$$x = 2$$

$$\therefore (2, 12)$$

$$f''(x) = 2 + 32x^{-3}$$

$$f''(2) = 6$$

$$f''(x) > 0 \therefore \text{minimum}$$

4 a $\text{area} = (2 \times \frac{1}{2}r^2\theta) + \frac{1}{2}r^2(3\theta) = 25$

$$\therefore \frac{5}{2}r^2\theta = 25, \theta = \frac{10}{r^2}$$

b $P = 2r + (2 \times r\theta) + r(3\theta) = 2r + 5r\theta$

$$= 2r + 5r\left(\frac{10}{r^2}\right) = 2r + \frac{50}{r}$$

c $\frac{dP}{dr} = 2 - 50r^{-2}$

SP: $2 - 50r^{-2} = 0$

$$r^2 = 25$$

$$r = 5$$

d $\min P = 20$

e $\frac{d^2P}{dr^2} = 100r^{-3}$, when $r = 5$, $\frac{d^2P}{dr^2} = 0.8$

$$\frac{d^2P}{dr^2} > 0 \therefore \text{minimum}$$

5 a $2x - x^{\frac{3}{2}} = 0$

$$x(2 - x^{\frac{1}{2}}) = 0$$

$$x = 0 \text{ or } x^{\frac{1}{2}} = 2 \Rightarrow x = 4$$

$$\therefore (0, 0) \text{ and } (4, 0)$$

b $\frac{dy}{dx} = 2 - \frac{3}{2}x^{\frac{1}{2}}$

SP: $2 - \frac{3}{2}x^{\frac{1}{2}} = 0$

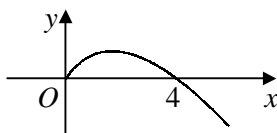
$$x^{\frac{1}{2}} = \frac{4}{3}$$

$$x = \frac{16}{9}$$

$$\frac{d^2y}{dx^2} = -\frac{3}{4}x^{-\frac{1}{2}}, \text{ when } x = \frac{16}{9}, \frac{d^2y}{dx^2} = -\frac{9}{16}$$

$$\frac{d^2y}{dx^2} < 0 \therefore \text{maximum}$$

c



6 a $\frac{dy}{dx} = 3x^2 - 3$

SP: $3x^2 - 3 = 0$

$$x^2 = 1$$

$$x = \pm 1$$

$$\therefore (-1, 3) \text{ and } (1, -1)$$

b $PQ^2 = 2^2 + 4^2 = 20$

$$\therefore PQ = \sqrt{20} = 2\sqrt{5}$$

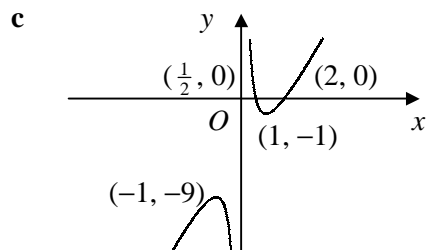
7 a $2x - 5 + \frac{2}{x} = 0$

$$2x^2 - 5x + 2 = 0$$

$$(2x - 1)(x - 2) = 0$$

$$x = \frac{1}{2}, 2$$

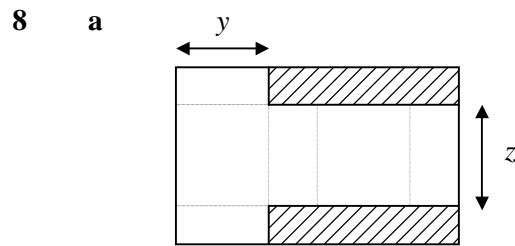
b $f'(x) = 2 - 2x^{-2}$
 $\therefore 2 - 2x^{-2} = 0$
 $x^2 = 1$
 $x = \pm 1$



9 a $\frac{dy}{dx} = 9 + 6x - 3x^2$
 SP: $9 + 6x - 3x^2 = 0$
 $-3(x + 1)(x - 3) = 0$
 $x = -1, 3$
 $\therefore (-1, -3)$ and $(3, 29)$

b $\frac{d^2y}{dx^2} = 6 - 6x$
 $(-1, -3): \frac{d^2y}{dx^2} = 12 \therefore$ minimum
 $(3, 29): \frac{d^2y}{dx^2} = -12 \therefore$ maximum

c $-3 < k < 29$



$$2x + z = 25$$

$$2x + 2y = 40$$

\therefore length and width $(25 - 2x)$ and $(20 - x)$

b volume = $x(25 - 2x)(20 - x)$
 $= x(500 - 65x + 2x^2)$
 $= 2x^3 - 65x^2 + 500x$

c $\frac{dV}{dx} = 6x^2 - 130x + 500$
 SP: $6x^2 - 130x + 500 = 0$
 $2(3x - 50)(x - 5) = 0$
 $x = 5, \frac{50}{3}$

$$2x < 25 \therefore x < 12.5$$

$$\therefore x = 5$$

d max volume = 1125 cm^3

$$\frac{d^2V}{dx^2} = 12x - 130$$

when $x = 5, \frac{d^2V}{dx^2} = -70$

$$\frac{d^2V}{dx^2} < 0 \therefore \text{maximum}$$

10 a $f(-1) = 15$
 $\therefore -4 + a + 12 + b = 15$
 $a + b = 7 \quad (1)$

b $f(2) = 42$
 $\therefore 32 + 4a - 24 + b = 42$
 $4a + b = 34 \quad (2)$

$$(2) - (1) \quad 3a = 27$$

$$\therefore a = 9, b = -2$$

c $f(x) = 4x^3 + 9x^2 - 12x - 2$
 $f'(x) = 12x^2 + 18x - 12$
 SP: $12x^2 + 18x - 12 = 0$
 $2x^2 + 3x - 2 = 0$
 $(2x - 1)(x + 2) = 0$
 $x = -2, \frac{1}{2}$
 $\therefore (-2, 26)$ and $(\frac{1}{2}, -\frac{21}{4})$