

- 1 Given that $a = \log_{10} 2$ and $b = \log_{10} 3$, find expressions in terms of a and b for
- a $\log_{10} 1.5$, (2)
- b $\log_{10} 24$, (2)
- c $\log_{10} 150$. (3)

- 2 Find, to an appropriate degree of accuracy, the values of x for which
- a $4 \log_3 x - 5 = 0$, (2)
- b $\log_3 x^3 - 5 \log_3 x = 4$. (3)

- 3 a Given that $p = \log_2 q$, find expressions in terms of p for
- i $\log_2 \sqrt{q}$,
- ii $\log_2 8q$. (4)
- b Solve the equation

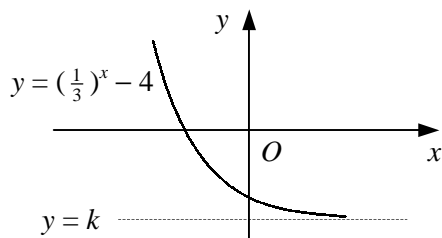
$$\log_2 8q - \log_2 \sqrt{q} = \log_3 9. \quad (3)$$

- 4 An initial investment of £1000 is placed into a savings account that offers 2.2% interest every 3 months. The amount of money in the account, £ P , at the end of t years is given by

$$P = 1000 \times 1.022^{4t}$$

Find, to the nearest year, how long it will take for the investment to double in value. (4)

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The diagram shows the curve with equation $y = (\frac{1}{3})^x - 4$.

- a Write down the coordinates of the point where the curve crosses the y -axis. (1)
- The curve has an asymptote with equation $y = k$.
- b Write down the value of the constant k . (1)
- c Find the x -coordinate of the point where the curve crosses the x -axis. (3)

- 6 a Solve the equation
- $$\log_3 (x + 1) - \log_3 (x - 2) = 1. \quad (3)$$

- b Find, in terms of logarithms to the base 10, the exact value of x such that
- $$3^{2x+1} = 2^{x-4}. \quad (3)$$

- 7 a Given that $t = 2^x$, write down expressions in terms of t for
- i 2^{x-1} ,
- ii 2^{2x+1} . (3)

- b Hence solve the equation
- $$2^{2x+1} - 14(2^{x-1}) + 6 = 0. \quad (5)$$

- 8** Find the values of x for which
- a** $\log_2(3x + 5) + \log_5 125 = 7$, (3)
- b** $\log_2(x + 1) = 5 - \log_2(3x - 1)$. (5)
- 9** Given that $\log_a(x + 4) = \log_a \frac{x}{4} + \log_a 5$,
and that $\log_a(y + 2) = \log_a 12 - \log_a(y + 1)$,
where $y > 0$, find
- a** the value of x , (3)
- b** the value of y , (4)
- c** the value of the logarithm of x to the base y . (2)
- 10** A colony of fast-breeding fish is introduced into a large, newly-built pond. The number of fish in the pond, n , after t weeks is modelled by
- $$n = \frac{18000}{1 + 8c^{-t}}.$$
- a** Find the initial number of fish in the pond. (2)
- Given that there are 3600 fish in the pond after 3 weeks, use this model to
- b** show that $c = \sqrt[3]{2}$, (3)
- c** find the time taken for the initial population of fish to double in size, giving your answer to the nearest day. (4)
- 11** **a** Given that $y = \log_8 x$, find expressions in terms of y for
- i** $\log_8 x^2$,
- ii** $\log_2 x$. (4)
- b** Hence, or otherwise, find the value of x such that
- $$3 \log_8 x^2 + \log_2 x = 6. \quad (3)$$
- 12** Solve the simultaneous equations
- $$\log_2 y = \log_2(3 - 2x) + 1$$
- $$\log_4 x + \log_4 y = \frac{1}{2} \quad (8)$$
- 13** **a** Sketch on the same diagram the curves $y = 2^x + 1$ and $y = (\frac{1}{2})^x$, showing the coordinates of any points where each curve meets the coordinate axes. (4)
- Given that the curves $y = 2^x + 1$ and $y = (\frac{1}{2})^x$ intersect at the point A ,
- b** show that the x -coordinate of A is a solution of the equation
- $$2^{2x} + 2^x - 1 = 0, \quad (2)$$
- c** hence, show that the y -coordinate of A is $\frac{1}{2}(\sqrt{5} + 1)$. (4)
- 14** **a** Show that $x = 1$ is a solution of the equation
- $$2^{3x} - 4(2^{2x}) + 2^x + 6 = 0. \quad (I) \quad (1)$$
- b** Show that using the substitution $u = 2^x$, equation (I) can be written as
- $$u^3 - 4u^2 + u + 6 = 0. \quad (2)$$
- c** Hence find the other real solution of equation (I) correct to 3 significant figures. (7)