

$$1 \quad \text{a} \quad f(x) = -[x^2 - 4x] + 3 \\ = -[(x-2)^2 - 4] + 3 \\ = -(x-2)^2 + 7$$

$$\therefore a = -1, b = -2, c = 7$$

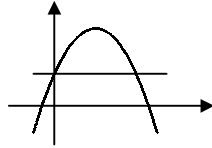
$$\text{b} \quad (2, 7)$$

c intersect when

$$3 + 4x - x^2 = 3$$

$$x(4-x) = 0$$

$$x = 0, 4$$



area below curve

$$= \int_0^4 (3 + 4x - x^2) dx$$

$$= [3x + 2x^2 - \frac{1}{3}x^3]_0^4$$

$$= (12 + 32 - \frac{64}{3}) - 0 = \frac{68}{3}$$

area below line

$$= 4 \times 3 = 12$$

area between line and curve

$$= \frac{68}{3} - 12 = 10\frac{2}{3}$$

$$3 \quad \text{a} \quad \frac{dy}{dx} = 5 - 4x$$

$$\text{grad} = 1$$

$$\therefore \text{grad of normal} = -1$$

$$\therefore y - 3 = -(x - 1)$$

$$[y = 4 - x]$$

b area below curve

$$= \int_0^1 (5x - 2x^2) dx$$

$$= [\frac{5}{2}x^2 - \frac{2}{3}x^3]_0^1$$

$$= (\frac{5}{2} - \frac{2}{3}) - 0 = \frac{11}{6}$$

normal meets y-axis at (0, 4)

area below line

$$= \frac{1}{2} \times 1 \times (4 + 3) = \frac{7}{2}$$

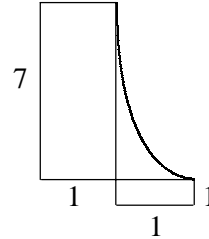
shaded area

$$= \frac{7}{2} - \frac{11}{6} = \frac{5}{3}$$

$$2 \quad \text{a} \quad = [-4x^{-2}]_1^2 \\ = -1 - (-4) \\ = 3$$

$$\text{b} \quad y = 1 \Rightarrow x = 2$$

$$y = 8 \Rightarrow x = 1$$



shaded area

$$= 3 - (1 \times 1) + (7 \times 1)$$

$$= 9$$

$$4 \quad \text{a} \quad \frac{4-x^2}{x^2} = 0$$

$$4 - x^2 = 0$$

$$x^2 = 4$$

$$x > 0 \therefore x = 2, P(2, 0)$$

$$\text{b} \quad l: y - 0 = -3(x - 2)$$

$$y = 6 - 3x$$

$$\text{intersect when } \frac{4-x^2}{x^2} = 6 - 3x$$

$$4 - x^2 = 6x^2 - 3x^3$$

$$3x^3 - 7x^2 + 4 = 0$$

$$x = 2 \text{ is a solution } \therefore (x - 2) \text{ is a factor}$$

$$(x - 2)(3x^2 - x - 2) = 0$$

$$(x - 2)(3x + 2)(x - 1) = 0$$

$$x = 2 \text{ (at } P), -\frac{2}{3}, 1$$

$$x > 0 \therefore Q(1, 3)$$

c area below curve

$$= \int_1^2 (4x^{-2} - 1) dx$$

$$= [-4x^{-1} - x]_1^2$$

$$= (-2 - 2) - (-4 - 1) = 1$$

area below line

$$= \frac{1}{2} \times 1 \times 3 = \frac{3}{2}$$

area between line and curve

$$= \frac{3}{2} - 1 = \frac{1}{2}$$