C2

INTEGRATION

Worksheet D

1 Evaluate

a
$$\int_{1}^{4} \frac{2}{x^{2}} dx$$
, (3)

b
$$\int_0^2 (x-3)^2 dx$$
. (4)

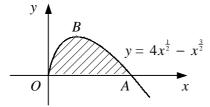
3

$$f(x) \equiv 3x^{\frac{1}{2}} - x^{-\frac{1}{2}}.$$

a Find the value of f(2), giving your answer in the form $k\sqrt{2}$ where k is an exact fraction. (2)

b Show that
$$\int_3^4 f(x) dx = 12 - 4\sqrt{3}$$
. (4)

4



The diagram shows the curve with the equation $y = 4x^{\frac{1}{2}} - x^{\frac{3}{2}}$.

The curve meets the x-axis at the origin, O, and at the point A.

a Find the coordinates of the point A.

(2)

The curve has a maximum at the point B.

b Find the *x*-coordinate of the point *B*.

(5)

c Find the area of the shaded region enclosed by the curve and the *x*-axis.

(4)

The curve $y = 4 + \frac{1}{x}$ crosses the x-axis at the point (p, 0) and has an asymptote y = q.

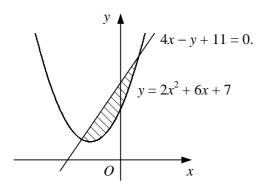
a Write down the values of p and q.

(2)

b Sketch the curve.

(2)

6



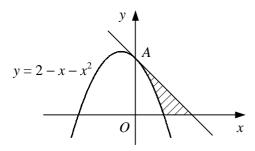
The diagram shows the curve with the equation $y = 2x^2 + 6x + 7$ and the straight line with the equation 4x - y + 11 = 0.

- a Find the coordinates of the points where the curve and line intersect. (5)
- **b** Find the area of the shaded region enclosed by the curve and the line. (6)

8 **a** Expand $(1 + \frac{x}{10})^{12}$ in ascending powers of x up to and including the term in x^3 , simplifying each coefficient in the expansion. (4)

b Using your series expansion from part **a**, find an estimate for $\int_0^1 (1 + \frac{x}{10})^{12} dx$. (5)

9



The diagram shows the curve with the equation $y = 2 - x - x^2$ and the tangent to the curve at the point A where it crosses the y-axis.

a Find an equation of the tangent to the curve at A. (4)

b Show that the area of the shaded region enclosed by the curve, the tangent to the curve at *A* and the *x*-axis is $\frac{5}{6}$. (9)