

- 1** **a** $r = 3$
 $u_8 = 3 \times 3^7 = 6561$
- b** $r = \frac{1}{4}$
 $u_8 = 1024 \times (\frac{1}{4})^7 = \frac{1}{16}$
- c** $r = -2$
 $u_8 = 1 \times (-2)^7 = -128$
- 2** **a** $a = 1, r = 5$
 $u_n = 5^{n-1}$
- b** $a = 3, r = -4$
 $u_n = 3 \times (-4)^{n-1}$
- c** $a = 81, r = \frac{2}{3}$
 $u_n = 81 \times (\frac{2}{3})^{n-1}$
- 3** **a** $a = 2, r = 2, n = 12$
 $S_{12} = \frac{2(2^{12} - 1)}{2 - 1} = 8190$
- b** $a = 640, r = \frac{1}{2}, n = 12$
 $S_{12} = \frac{640[1 - (\frac{1}{2})^{12}]}{1 - \frac{1}{2}} = 1279\frac{11}{16}$
- c** $a = \frac{1}{6}, r = -3, n = 12$
 $S_{12} = \frac{\frac{1}{6}[1 - (-3)^{12}]}{1 - (-3)} = -22\,143\frac{1}{3}$
- 4** **a** $S_8 = \frac{4(3^8 - 1)}{3 - 1} = 13\,120$
- b** $S_{14} = \frac{48[1 - (\frac{1}{2})^{14}]}{1 - \frac{1}{2}} = 95.994$
- c** $S_{12} = \frac{-[1 - (-4)^{12}]}{1 - (-4)} = 3\,355\,443$
- d** $S_{20} = \frac{200[1 - (0.7)^{20}]}{1 - 0.7} = 666.135$
- e** $S_{15} = \frac{120[1 - (\frac{3}{4})^{15}]}{1 - (\frac{3}{4})} = 69.488$
- f** $S_{30} = \frac{-25[(1.2)^{30} - 1]}{1.2 - 1} = -29\,547.039$
- 5** **a** GP: $a = 3$
 $r = 3, n = 9$
 $S_9 = \frac{3(3^9 - 1)}{3 - 1} = 29\,523$
- b** GP: $a = 64$
 $r = 8, n = 6$
 $S_6 = \frac{64(8^6 - 1)}{8 - 1} = 2\,396\,736$
- c** GP: $a = 20$
 $r = 2, n = 10$
 $S_{10} = \frac{20(2^{10} - 1)}{2 - 1} = 20\,460$
- d** GP: $a = 0.8$
 $r = 0.8, n = 8$
 $S_8 = \frac{0.8[1 - (0.8)^8]}{1 - 0.8} = 3.329$ (3dp)
- e** GP: $a = 2$
 $r = \frac{1}{6}, n = 10$
 $S_{10} = \frac{2[1 - (\frac{1}{6})^{10}]}{1 - \frac{1}{6}} = 2.400$ (3dp)
- f** GP: $a = -4$
 $r = -4, n = 9$
 $S_9 = \frac{-4[1 - (-4)^9]}{1 - (-4)} = -209\,716$
- g** GP: $a = \frac{1}{16}$
 $r = \frac{1}{2}, n = 17$
 $S_{17} = \frac{\frac{1}{16}[1 - (\frac{1}{2})^{17}]}{1 - \frac{1}{2}} = 0.125$ (3dp)
- h** GP: $a = -54$
 $r = -3, n = 7$
 $S_7 = \frac{-54[1 - (-3)^7]}{1 - (-3)} = -29\,538$
- 6** **a** $r = 10 \div 2 = 5$
- b** $a \times 5 = 2 \therefore a = 0.4$
- c** $S_8 = \frac{0.4(5^8 - 1)}{5 - 1} = 39\,062.4$
- 7** **a** $a = 2, ar^3 = 54 \therefore r^3 = 54 \div 2 = 27$
 $r = \sqrt[3]{27} = 3$
- b** $u_9 = 2 \times 3^8 = 13\,122$
- 8** **a** $r = 8 \div 24 = \frac{1}{3}$
- b** $a \times (\frac{1}{3})^2 = 24 \therefore a = 216$
- c** $S_{11} = \frac{216[1 - (\frac{1}{3})^{11}]}{1 - \frac{1}{3}} = 323.998$
- 9** **a** $a = 6, ar^2 = 24 \therefore r^2 = 24 \div 6 = 4$
 $r = \pm 2$
- b** $r = 2, S_{15} = \frac{6(2^{15} - 1)}{2 - 1} = 196\,602$
- 10** **a** $a = 768, ar^3 = -96$
 $r^3 = -96 \div 768 = -\frac{1}{8}$
 $r = \sqrt[3]{-\frac{1}{8}} = -\frac{1}{2}$
- b** $u_{10} = 768 \times (-\frac{1}{2})^9 = -1.5$
- 11** **a** $ar = 0.5, ar^4 = 32 \therefore r^3 = 32 \div 0.5 = 64$
 $r = \sqrt[3]{64} = 4, a \times 4 = 0.5 \therefore a = 0.125$
- b** $0.125 \times 4^{n-1} < 10\,000 \therefore 4^{n-1} < 80\,000$
 $(n - 1) \lg 4 < \lg 80\,000$
 $n < \frac{\lg 80\,000}{\lg 4} + 1$
 $n < 9.14 \therefore 9$ terms

$$12 \quad \mathbf{a} \quad \frac{a[(\frac{3}{2})^4 - 1]}{\frac{3}{2} - 1} = 130$$

$$a = 130 \div \frac{65}{8} = 16$$

$$\mathbf{b} \quad u_8 = 16 \times (\frac{3}{2})^7 = 273\frac{3}{8}$$

$$\mathbf{c} \quad \frac{16[(\frac{3}{2})^n - 1]}{\frac{3}{2} - 1} > 30\,000$$

$$(\frac{3}{2})^n > 938.5$$

$$n \lg \frac{3}{2} > \lg 938.5$$

$$n > \frac{\lg 938.5}{\lg 1.5}$$

$$n > 16.9 \quad \therefore \text{least } n = 17$$

$$13 \quad \mathbf{a} \quad a + ar = a(1 + r) = 10.8$$

$$ar^2 + ar^3 = ar^2(1 + r) = 43.2$$

$$\therefore r^2 = 43.2 \div 10.8 = 4$$

all terms +ve $\therefore r + \text{ve} \therefore r = 2$

$$\text{sub. } a = 10.8 \div 3 = 3.6$$

$$\mathbf{b} \quad S_{16} = \frac{3.6(2^{16} - 1)}{2 - 1} = 235\,926$$

$$14 \quad \mathbf{a} \quad a = 12, r = 0.5$$

$$S_{\infty} = \frac{12}{1 - 0.5} = 24$$

$$\mathbf{b} \quad a = 270, r = \frac{1}{3}$$

$$S_{\infty} = \frac{270}{1 - \frac{1}{3}} = 405$$

$$\mathbf{c} \quad a = 25, r = -1.2$$

no S_{∞} as $r < -1 \therefore$ diverges

$$\mathbf{d} \quad a = 216, r = \frac{2}{3}$$

$$S_{\infty} = \frac{216}{1 - \frac{2}{3}} = 648$$

$$\mathbf{e} \quad a = \frac{8}{25}, r = \frac{5}{4}$$

no S_{∞} as $r > 1 \therefore$ diverges

$$\mathbf{f} \quad a = 500, r = -0.6$$

$$S_{\infty} = \frac{500}{1 - (-0.6)} = 312.5$$

$$15 \quad \mathbf{a} \quad a = 0.9, r = 0.9$$

$$S_{\infty} = \frac{0.9}{1 - 0.9} = 9$$

$$\mathbf{b} \quad a = 3, r = \frac{1}{2}$$

$$S_{\infty} = \frac{3}{1 - \frac{1}{2}} = 6$$

$$\mathbf{c} \quad a = 1, r = -\frac{3}{4}$$

$$S_{\infty} = \frac{1}{1 - (-\frac{3}{4})} = \frac{4}{7}$$

$$\mathbf{d} \quad a = 32, r = 0.8$$

$$S_{\infty} = \frac{32}{1 - 0.8} = 160$$

$$16 \quad \mathbf{a} \quad S_{\infty} = \frac{80}{1 - 0.2} = 100$$

$$\mathbf{b} \quad S_6 = \frac{80[1 - (0.2)^6]}{1 - 0.2} = 99.9936$$

$$S_{\infty} - S_6 = 0.0064$$

$$17 \quad \mathbf{a} \quad 1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}$$

$$\mathbf{b} \quad \text{GP: } a = 1, r = \frac{1}{3}$$

$$S_{\infty} = \frac{1}{1 - \frac{1}{3}} = \frac{3}{2}$$

$$18 \quad \mathbf{a} \quad \frac{a}{1 - 0.55} = 40$$

$$a = 0.45 \times 40 = 18$$

$$\mathbf{b} \quad 18 \times (0.55)^{n-1} < 0.001$$

$$(n - 1) \lg 0.55 < \lg 0.0000556$$

$$n > \frac{\lg 0.0000556}{\lg 0.55} + 1$$

$$n > 17.4 \quad \therefore \text{smallest } n = 18$$

$$19 \quad \mathbf{a} \quad u_1 = S_1 = 2^1 - 1 = 1$$

$$S_5 = 2^5 - 1 = 31, S_4 = 2^4 - 1 = 15$$

$$u_5 = S_5 - S_4 = 31 - 15 = 16$$

$$\mathbf{b} \quad S_{n-1} = 2^{n-1} - 1$$

$$u_n = S_n - S_{n-1} = (2^n - 1) - (2^{n-1} - 1)$$

$$= 2^n - 2^{n-1} = 2^{n-1}(2 - 1) = 2^{n-1}$$

$$20 \quad \mathbf{a} \quad \frac{k}{k+10} = \frac{k-6}{k}$$

$$k^2 = (k+10)(k-6)$$

$$4k - 60 = 0$$

$$k = 15$$

$$\mathbf{b} \quad u_1 = 25, u_2 = 15 \quad \therefore a = 25, r = 0.6$$

$$S_{\infty} = \frac{25}{1 - 0.6} = 62.5$$