C2 SEQUENCES AND SERIES Worksheet B

1 The third and fourth terms of a geometric series are 27 and $20\frac{1}{4}$ respectively.

a Find the first term of the series.

b Find the sum to infinity of the series.

2 The first three terms of a geometric series are $(k - 8)$, $(k + 4)$ and $(3k + 2)$ respectively, where k is a positive constant.

a Find the value of k.

b Find the sixth term of the series.

c Show that the sum of the first ten terms of the series is 50 857.3 to 1 decimal place.

3 The second and fifth terms of a geometric series are 75 and 129.6 respectively.

a Show that the first term of the series is 62.5

b Find the value of the tenth term of the series to 1 decimal place.

c Find the sum of the first 12 terms of the series to 1 decimal place.

4 **a** Prove that the sum, S_n, of the first n terms of a geometric series with first term a and common ratio r is given by

$$S_n = \frac{a(1-r^n)}{1-r}.$$

b A geometric series has first term 2 and common ratio $\sqrt{2}$.

Given that the sum of the first n terms of the series is $126(\sqrt{2} + 1)$, find the value of n.

5 The first term of a geometric series is 18 and the sum to infinity of the series is 15.

a Find the common ratio of the series.

b Find the third term of the series.

c Find the exact difference between the sum of the first eight terms of the series and the sum to infinity of the series.

6 The sum of the first n terms of a geometric series is given by $5(3^n - 1)$.

a Show that the third term of the series is 90.

b Find an expression for the nth term of the series in the form $k(3^n)$ where k is an exact fraction.

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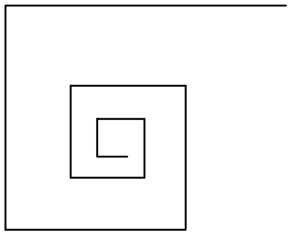

A student programs a computer to draw a series of straight lines with each line beginning at the end of the previous one and at right angles to it. The first line is 4 mm long and thereafter each line is 25% longer than the previous one, so that a spiral is formed as shown above.

a Find the length, in mm, of the eighth straight line drawn by the program.

b Find the total length of the spiral, in metres, when 20 straight lines have been drawn.

8 The second and fourth terms of a geometric series are 30 and 2.7 respectively.

Given that the common ratio, r, of the series is positive,

a find the value of r and the first term of the series,

b find the sum to infinity of the series.

9 **a** Evaluate $\sum_{r=3}^{10} 3^r$.

b Show that $\sum_{r=1}^{15} (2^r - 12r) = 64\,094$.

10 A geometric series has common ratio r and the nth term of the series is denoted by u_n.

Given that $u_1 = 64$ and that $u_3 - u_2 = 20$,

a show that $16r^2 - 16r - 5 = 0$,

b find the two possible values of r,

c find the fourth term of the series corresponding to each possible value of r.

d Taking the value of r such that the series converges, find the sum to infinity of the series.

11 A geometric series has first term 4 and common ratio $\frac{1}{2}$.

a Find the eighth term of the series as an exact fraction.

b Find the nth term of the series in the form 2^y where y is a function of n.

c Show that the sum of the first n terms of the series is $8 - 2^{3-n}$.

12 The sequence of terms $u_1, u_2, u_3, \ldots$ is defined by

$$u_n = 4 \times 3^n, \quad n \geq 1.$$

a Find u_6 .

b Find the smallest value of t such that the sum of the first t terms of the sequence is greater than 10^{25}.

13 The sum of the first and third terms of a geometric series is 150. The sum of the second and fourth terms of the series is −75.

a Find the first term and common ratio of the series.

b Find the sum to infinity of the series.

14 Three consecutive terms of an arithmetic series are a, b and $(3a + 4)$ respectively.

a Find an expression for b in terms of a.

Given also that a, b and $(6a + 1)$ respectively are consecutive terms of a geometric series and that a and b are integers,

b find the values of a and b.

15 When a ball is dropped onto a horizontal floor it bounces such that it reaches a maximum height of 60% of the height from which it was dropped.

a Find the maximum height the ball reaches after its fourth bounce when it is initially dropped from 3 metres above the floor.

b Show that when the ball is dropped from a height of h metres above the floor it travels a total distance of $4h$ metres before coming to rest.