

- 1
- a**  $= 1 + 4(3x) + 6(3x)^2 + 4(3x)^3 + (3x)^4$   
 $= 1 + 12x + 54x^2 + 108x^3 + 81x^4$
- b**  $= 2^5 + 5(2^4)(-x) + 10(2^3)(-x)^2 + 10(2^2)(-x)^3 + 5(2)(-x)^4 + (-x)^5$   
 $= 32 - 80x + 80x^2 - 40x^3 + 10x^4 - x^5$
- c**  $= 3^3 + 3(3^2)(10x^2) + 3(3)(10x^2)^2 + (10x^2)^3$   
 $= 27 + 270x^2 + 900x^4 + 1000x^6$
- d**  $= a^5 + 5a^4(2b) + 10a^3(2b)^2 + 10a^2(2b)^3 + 5a(2b)^4 + (2b)^5$   
 $= a^5 + 10a^4b + 40a^3b^2 + 80a^2b^3 + 80ab^4 + 32b^5$
- e**  $= (x^2)^3 + 3(x^2)^2(-y) + 3(x^2)(-y)^2 + (-y)^3$   
 $= x^6 - 3x^4y + 3x^2y^2 - y^3$
- f**  $= 5^4 + 4(5^3)(\frac{1}{2}x) + 6(5^2)(\frac{1}{2}x)^2 + 4(5)(\frac{1}{2}x)^3 + (\frac{1}{2}x)^4$   
 $= 625 + 250x + \frac{75}{2}x^2 + \frac{5}{2}x^3 + \frac{1}{16}x^4$
- g**  $= x^4 + 4x^3(\frac{1}{x}) + 6x^2(\frac{1}{x})^2 + 4x(\frac{1}{x})^3 + (\frac{1}{x})^4$   
 $= x^4 + 4x^2 + 6 + \frac{4}{x^2} + \frac{1}{x^4}$
- h**  $= t^3 + 3t^2(-\frac{2}{t^2}) + 3t(-\frac{2}{t^2})^2 + (-\frac{2}{t^2})^3$   
 $= t^3 - 6 + \frac{12}{t^3} - \frac{8}{t^6}$
- 2
- a**  $= 1 + 6(3x) + \frac{6 \times 5}{2}(3x)^2 + \frac{6 \times 5 \times 4}{3 \times 2}(3x)^3 + \dots$   
 $= 1 + 18x + 135x^2 + 540x^3 + \dots$
- b**  $= 1 + 8(-\frac{1}{4}x) + \frac{8 \times 7}{2}(-\frac{1}{4}x)^2 + \frac{8 \times 7 \times 6}{3 \times 2}(-\frac{1}{4}x)^3 + \dots$   
 $= 1 - 2x + \frac{7}{4}x^2 - \frac{7}{8}x^3 + \dots$
- c**  $= 5^7 + 7(5^6)(-x) + \frac{7 \times 6}{2}(5^5)(-x)^2 + \frac{7 \times 6 \times 5}{3 \times 2}(5^4)(-x)^3 + \dots$   
 $= 78\,125 - 109\,375x + 65\,625x^2 - 21\,875x^3 + \dots$
- d**  $= 3^{10} + 10(3^9)(2x^2) + \frac{10 \times 9}{2}(3^8)(2x^2)^2 + \frac{10 \times 9 \times 8}{3 \times 2}(3^7)(2x^2)^3 + \dots$   
 $= 59\,049 + 393\,660x^2 + 1\,180\,980x^4 + 2\,099\,520x^6 + \dots$
- 3
- a**  $= \binom{15}{3} = 455$
- b**  $= \binom{12}{4} \times (-2)^4 = 7920$
- c**  $= \binom{7}{2} \times 3^5 = 5103$
- d**  $= \binom{10}{5} \times 2^5 \times (-1)^5 = -8064$
- e**  $= \binom{8}{5} \times 2^3 = 448$
- f**  $= \binom{9}{3} \times (-1)^3 = -84$
- 4
- a**  $= (\sqrt{2})^4 + 4(\sqrt{2})^3(-\sqrt{5}) + 6(\sqrt{2})^2(-\sqrt{5})^2 + 4(\sqrt{2})(-\sqrt{5})^3 + (-\sqrt{5})^4$   
 $= 4 - 8\sqrt{10} + 60 - 20\sqrt{10} + 25$   
 $= 89 - 28\sqrt{10}$
- b**  $= (\sqrt{2})^3 + 3(\sqrt{2})^2(\frac{1}{\sqrt{3}}) + 3(\sqrt{2})(\frac{1}{\sqrt{3}})^2 + (\frac{1}{\sqrt{3}})^3$   
 $= 2\sqrt{2} + 2\sqrt{3} + \sqrt{2} + \frac{1}{9}\sqrt{3}$   
 $= 3\sqrt{2} + \frac{19}{9}\sqrt{3}$

$$\begin{aligned}
 \mathbf{c} &= 1 + 3(\sqrt{5}) + 3(\sqrt{5})^2 + (\sqrt{5})^3 - [1 + 3(-\sqrt{5}) + 3(-\sqrt{5})^2 + (-\sqrt{5})^3] \\
 &= 1 + 3\sqrt{5} + 15 + 5\sqrt{5} - [1 - 3\sqrt{5} + 15 - 5\sqrt{5}] \\
 &= 16 + 8\sqrt{5} - [16 - 8\sqrt{5}] \\
 &= 16\sqrt{5}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{5} \quad \mathbf{a} &= 1 + 10\left(\frac{x}{2}\right) + \frac{10 \times 9}{2} \left(\frac{x}{2}\right)^2 + \frac{10 \times 9 \times 8}{3 \times 2} \left(\frac{x}{2}\right)^3 + \dots \\
 &= 1 + 5x + \frac{45}{4}x^2 + 15x^3 + \dots
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad \mathbf{i} \quad \text{let } x &= 0.01 \\
 1.005^{10} &\approx 1 + 0.05 + 0.001\,125 + 0.000\,015 \\
 &= 1.051\,14 \text{ (5dp)}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{ii} \quad \text{let } x &= -0.008 \\
 0.996^{10} &\approx 1 - 0.040 + 0.000\,720 - 0.000\,007\,680 \\
 &= 0.960\,71 \text{ (5dp)}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{6} \quad \mathbf{a} &= 3^8 + 8(3^7)x + \frac{8 \times 7}{2} (3^6)x^2 + \frac{8 \times 7 \times 6}{3 \times 2} (3^5)x^3 + \dots \\
 &= 6561 + 17\,496x + 20\,412x^2 + 13\,608x^3 + \dots
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad \mathbf{i} \quad \text{let } x &= 0.001 \\
 3.001^8 &\approx 6561 + 17.496 + 0.020\,412 + 0.000\,013\,608 \\
 &= 6578.516 \text{ (7sf)}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{ii} \quad \text{let } x &= -0.005 \\
 2.995^8 &\approx 6561 - 87.480 + 0.510\,300 - 0.001\,701\,000 \\
 &= 6474.029 \text{ (7sf)}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{7} \quad \mathbf{a} \quad (1 + 10x)^4 &= 1 + 4(10x) + 6(10x)^2 + 4(10x)^3 + (10x)^4 \\
 &= 1 + 40x + 600x^2 + 4000x^3 + 10\,000x^4 \\
 \therefore (1 + 10x)^4 + (1 - 10x)^4 &= 1 + 40x + 600x^2 + 4000x^3 + 10\,000x^4 + (1 - 40x + 600x^2 - 4000x^3 + 10\,000x^4) \\
 &= 2 + 1200x^2 + 20\,000x^4
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad (2 + \frac{1}{3}x)^3 &= 2^3 + 3(2^2)(\frac{1}{3}x) + 3(2)(\frac{1}{3}x)^2 + (\frac{1}{3}x)^3 \\
 &= 8 + 4x + \frac{2}{3}x^2 + \frac{1}{27}x^3 \\
 \therefore (2 - \frac{1}{3}x)^3 - (2 + \frac{1}{3}x)^3 &= 8 - 4x + \frac{2}{3}x^2 - \frac{1}{27}x^3 - (8 + 4x + \frac{2}{3}x^2 + \frac{1}{27}x^3) \\
 &= -8x - \frac{2}{27}x^3
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{c} &= (1 + 4y)(1 + 3y + 3y^2 + y^3) \\
 &= 1 + 3y + 3y^2 + y^3 + 4y + 12y^2 + 12y^3 + 4y^4 \\
 &= 1 + 7y + 15y^2 + 13y^3 + 4y^4
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{d} &= (1 - x)\left(1 + \frac{3}{x} + \frac{3}{x^2} + \frac{1}{x^3}\right) \\
 &= 1 + \frac{3}{x} + \frac{3}{x^2} + \frac{1}{x^3} - x - 3 - \frac{3}{x} - \frac{1}{x^2} \\
 &= -x - 2 + \frac{2}{x^2} + \frac{1}{x^3}
 \end{aligned}$$

$$\begin{aligned}
 8 \quad \mathbf{a} &= (1+x^2)[1+10(-3x)+\frac{10 \times 9}{2}(-3x)^2+\frac{10 \times 9 \times 8}{3 \times 2}(-3x)^3+\dots] \\
 &= (1+x^2)[1-30x+405x^2-3240x^3+\dots] \\
 &= 1-30x+405x^2-3240x^3+x^2-30x^3+\dots \\
 &= 1-30x+406x^2-3270x^3+\dots
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} &= (1-2x)[1+8x+\frac{8 \times 7}{2}x^2+\frac{8 \times 7 \times 6}{3 \times 2}x^3+\dots] \\
 &= (1-2x)[1+8x+28x^2+56x^3+\dots] \\
 &= 1+8x+28x^2+56x^3-2x-16x^2-56x^3+\dots \\
 &= 1+6x+12x^2+\dots
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{c} &= (1+x+x^2)[1+6(-x)+\frac{6 \times 5}{2}(-x)^2+\frac{6 \times 5 \times 4}{3 \times 2}(-x)^3+\dots] \\
 &= (1+x+x^2)[1-6x+15x^2-20x^3+\dots] \\
 &= 1-6x+15x^2-20x^3+x-6x^2+15x^3+x^2-6x^3+\dots \\
 &= 1-5x+10x^2-11x^3+\dots
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{d} &= (1+3x-x^2)[1+7(2x)+\frac{7 \times 6}{2}(2x)^2+\frac{7 \times 6 \times 5}{3 \times 2}(2x)^3+\dots] \\
 &= (1+3x-x^2)[1+14x+84x^2+280x^3+\dots] \\
 &= 1+14x+84x^2+280x^3+3x+42x^2+252x^3-x^2-14x^3+\dots \\
 &= 1+17x+125x^2+518x^3+\dots
 \end{aligned}$$

$$9 \quad \mathbf{a} = \binom{8}{4} \times y^4 \times \left(\frac{1}{y}\right)^4 = 70$$

$$\mathbf{b} = \binom{12}{6} \times (2y)^6 \times \left(-\frac{1}{2y}\right)^6 = 924$$

$$\mathbf{c} = \binom{6}{2} \times \left(\frac{1}{y}\right)^4 \times (y^2)^2 = 15$$

$$\mathbf{d} = \binom{9}{3} \times (3y)^6 \times \left(-\frac{1}{y^2}\right)^3 = -61\,236$$

$$10 \quad \mathbf{a} \quad \frac{n(n-1)}{2} \times \left(\frac{2}{5}\right)^2 = 1.6$$

$$n(n-1) = \frac{25}{2} \times 1.6 = 20$$

$$n^2 - n - 20 = 0$$

$$(n+4)(n-5) = 0$$

$$n > 0 \quad \therefore n = 5$$

$$\mathbf{b} = 5 \times \left(\frac{2}{5}\right)^4 = \frac{16}{125} \text{ or } 0.128$$

$$11 \quad \mathbf{a} \quad y_1 = (1-2x)[1+10x+\frac{10 \times 9}{2}x^2+\dots]$$

$$= 1+10x+45x^2-2x-20x^2+\dots$$

$$= 1+8x+25x^2+\dots$$

$$\therefore a = 25, b = 8, c = 1$$

$$\mathbf{b} \quad x = 0.2: \quad y_1 = 0.6 \times (1.2)^{10} = 3.71504$$

$$y_2 = (25 \times 0.04) + (8 \times 0.2) + 1 = 3.6$$

$$\% \text{ error} = \frac{3.71504 - 3.6}{3.71504} \times 100\% = 3.1\% \text{ (2sf)}$$

$$12 \quad \mathbf{a} \quad (1+px)^q = 1 + q(px) + \frac{q(q-1)}{2}(px)^2 + \dots$$

$$\therefore pq = -12 \text{ and } \frac{1}{2}p^2q(q-1) = 60$$

$$\text{sub. } p = -\frac{12}{q}$$

$$\Rightarrow \frac{72}{q}(q-1) = 60$$

$$72(q-1) = 60q$$

$$q = 6, p = -2$$

$$\mathbf{b} = \frac{6 \times 5 \times 4}{3 \times 2} \times (-2)^3 = -160$$

$$13 \quad \mathbf{a} = 3^{12} + 12(3^{11})\left(-\frac{x}{3}\right) + \frac{12 \times 11}{2}(3^{10})\left(-\frac{x}{3}\right)^2 + \frac{12 \times 11 \times 10}{3 \times 2}(3^9)\left(-\frac{x}{3}\right)^3 + \dots$$

$$= 531\,441 - 708\,588x + 433\,026x^2 - 160\,380x^3 + \dots$$

$$\mathbf{b} \text{ let } \frac{x}{3} = 0.002 \quad \therefore x = 0.006$$

$$2.998^{12} \approx 531\,441 - 4251.528 + 15.588\,936 - 0.034\,642\,080$$

$$= 527\,205.03 \text{ (2dp)}$$

$$14 \quad \mathbf{a} = 1 - 5x + 10x^2 - 10x^3 + 5x^4 - x^5$$

$$\mathbf{b} = 3 - 2\sqrt{3} + \sqrt{3} - 2 = 1 - \sqrt{3}$$

$$\mathbf{c} \quad \mathbf{i} = [(\sqrt{3} + 1)(\sqrt{3} - 2)]^5 = (1 - \sqrt{3})^5$$

$$= 1 - 5(\sqrt{3}) + 10(\sqrt{3})^2 - 10(\sqrt{3})^3 + 5(\sqrt{3})^4 - (\sqrt{3})^5$$

$$= 1 - 5\sqrt{3} + 30 - 30\sqrt{3} + 45 - 9\sqrt{3}$$

$$= 76 - 44\sqrt{3}$$

$$\mathbf{ii} = (\sqrt{3} + 1)(76 - 44\sqrt{3})$$

$$= 76\sqrt{3} - 132 + 76 - 44\sqrt{3}$$

$$= -56 + 32\sqrt{3}$$

$$15 \quad \mathbf{a} = 1 + 9\left(\frac{x}{2}\right) + \frac{9 \times 8}{2}\left(\frac{x}{2}\right)^2 + \frac{9 \times 8 \times 7}{3 \times 2}\left(\frac{x}{2}\right)^3 + \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2}\left(\frac{x}{2}\right)^4 + \dots$$

$$= 1 + \frac{9}{2}x + 9x^2 + \frac{21}{2}x^3 + \frac{63}{8}x^4 + \dots$$

$$\mathbf{b} = \frac{21}{2} - \left(-\frac{21}{2}\right) = 21$$

$$\mathbf{c} = \left(1 \times \frac{63}{8}\right) + \left(2 \times \frac{21}{2}\right) = 28\frac{7}{8}$$

$$16 \quad 10(x^3)^2\left(\frac{a}{x^2}\right)^3 = -80$$

$$a^3 = -8$$

$$a = -2$$

$$17 \quad \mathbf{a} \quad \left(1 + \frac{x}{k}\right)^n = 1 + n\left(\frac{x}{k}\right) + \frac{n(n-1)}{2}\left(\frac{x}{k}\right)^2 + \frac{n(n-1)(n-2)}{3 \times 2}\left(\frac{x}{k}\right)^3 + \dots$$

$$\therefore \frac{n(n-1)}{2k^2} = 3 \times \frac{n(n-1)(n-2)}{6k^3}$$

$$kn(n-1) = n(n-1)(n-2)$$

$$n(n-1)[k - (n-2)] = 0$$

$$n > 1 \quad \therefore k - (n-2) = 0$$

$$k = n - 2$$

$$\mathbf{b} \quad k = 7 - 2 = 5$$

$$\left(1 + \frac{x}{5}\right)^7 = 1 + 7\left(\frac{x}{5}\right) + \frac{7 \times 6}{2}\left(\frac{x}{5}\right)^2 + \frac{7 \times 6 \times 5}{3 \times 2}\left(\frac{x}{5}\right)^3 + \frac{7 \times 6 \times 5 \times 4}{4 \times 3 \times 2}\left(\frac{x}{5}\right)^4 + \dots$$

$$= 1 + \frac{7}{5}x + \frac{21}{25}x^2 + \frac{7}{25}x^3 + \frac{7}{125}x^4 + \dots$$