

1 Expand

a $(1 + 3x)^4$

b $(2 - x)^5$

c $(3 + 10x^2)^3$

d $(a + 2b)^5$

e $(x^2 - y)^3$

f $(5 + \frac{1}{2}x)^4$

g $(x + \frac{1}{x})^4$

h $(t - \frac{2}{t^2})^3$

2 Find the first four terms in the expansion in ascending powers of x of

a $(1 + 3x)^6$

b $(1 - \frac{1}{4}x)^8$

c $(5 - x)^7$

d $(3 + 2x^2)^{10}$

3 Find the coefficient indicated in the following expansions

a $(1 + x)^{15}$, coefficient of x^3

b $(1 - 2x)^{12}$, coefficient of x^4

c $(3 + x)^7$, coefficient of x^2

d $(2 - y)^{10}$, coefficient of y^5

e $(2 + t^3)^8$, coefficient of t^{15}

f $(1 - \frac{1}{x})^9$, coefficient of x^{-3}

4 **a** Express $(\sqrt{2} - \sqrt{5})^4$ in the form $a + b\sqrt{10}$, where $a, b \in \mathbb{Z}$.

b Express $(\sqrt{2} + \frac{1}{\sqrt{3}})^3$ in the form $a\sqrt{2} + b\sqrt{3}$, where $a, b \in \mathbb{Q}$.

c Express $(1 + \sqrt{5})^3 - (1 - \sqrt{5})^3$ in the form $a\sqrt{5}$, where $a \in \mathbb{Z}$.

5 **a** Expand $(1 + \frac{x}{2})^{10}$ in ascending powers of x up to and including the term in x^3 , simplifying each coefficient.

b By substituting a suitable value of x into your answer for part **a**, obtain an estimate for

i 1.005^{10}

ii 0.996^{10}

giving your answers to 5 decimal places.

6 **a** Expand $(3 + x)^8$ in ascending powers of x up to and including the term in x^3 , simplifying each coefficient.

b By substituting a suitable value of x into your answer for part **a**, obtain an estimate for

i 3.001^8

ii 2.995^8

giving your answers to 7 significant figures.

7 Expand and simplify

a $(1 + 10x)^4 + (1 - 10x)^4$

b $(2 - \frac{1}{3}x)^3 - (2 + \frac{1}{3}x)^3$

c $(1 + 4y)(1 + y)^3$

d $(1 - x)(1 + \frac{1}{x})^3$

8 Expand each of the following in ascending powers of x up to and including the term in x^3 .

a $(1 + x^2)(1 - 3x)^{10}$

b $(1 - 2x)(1 + x)^8$

c $(1 + x + x^2)(1 - x)^6$

d $(1 + 3x - x^2)(1 + 2x)^7$

9 Find the term independent of y in each of the following expansions.

a $(y + \frac{1}{y})^8$

b $(2y - \frac{1}{2y})^{12}$

c $(\frac{1}{y} + y^2)^6$

d $(3y - \frac{1}{y^2})^9$

- 10** The coefficient of x^2 in the binomial expansion of $(1 + \frac{2}{5}x)^n$, where n is a positive integer, is 1.6
- Find the value of n .
 - Use your value of n to find the coefficient of x^4 in the expansion.
- 11** Given that $y_1 = (1 - 2x)(1 + x)^{10}$ and $y_2 = ax^2 + bx + c$ and that when x is small, y_2 can be used as an approximation for y_1 ,
- find the values of the constants a , b and c ,
 - find the percentage error in using y_2 as an approximation for y_1 when $x = 0.2$
- 12** In the binomial expansion of $(1 + px)^q$, where p and q are constants and q is a positive integer, the coefficient of x is -12 and the coefficient of x^2 is 60 .
- Find
- the value of p and the value of q ,
 - the value of the coefficient of x^3 in the expansion.
- 13**
- Expand $(3 - \frac{x}{3})^{12}$ as a binomial series in ascending powers of x up to and including the term in x^3 , giving each coefficient as an integer.
 - Use your series expansion with a suitable value of x to obtain an estimate for 2.998^{12} , giving your answer to 2 decimal places.
- 14**
- Expand $(1 - x)^5$ as a binomial series in ascending powers of x .
 - Express $(\sqrt{3} + 1)(\sqrt{3} - 2)$ in the form $A + B\sqrt{3}$, where $A, B \in \mathbb{Z}$.
 - Hence express each of the following in the form $C + D\sqrt{3}$, where $C, D \in \mathbb{Z}$.
 - $(\sqrt{3} + 1)^5(\sqrt{3} - 2)^5$
 - $(\sqrt{3} + 1)^6(\sqrt{3} - 2)^5$
- 15**
- Expand $(1 + \frac{x}{2})^9$ in ascending powers of x up to and including the term in x^4 .
- Hence, or otherwise, find
- the coefficient of x^3 in the expansion of $(1 + \frac{x}{2})^9 - (1 - \frac{x}{2})^9$,
 - the coefficient of x^4 in the expansion of $(1 + 2x)(1 + \frac{x}{2})^9$.
- 16** The term independent of x in the expansion of $(x^3 + \frac{a}{x^2})^5$ is -80 .
- Find the value of the constant a .
- 17** In the binomial expansion of $(1 + \frac{x}{k})^n$, where k is a non-zero constant, n is an integer and $n > 1$, the coefficient of x^2 is three times the coefficient of x^3 .
- Show that $k = n - 2$.
- Given also that $n = 7$,
- expand $(1 + \frac{x}{k})^n$ in ascending powers of x up to and including the term in x^4 , giving each coefficient as a fraction in its simplest form.