2 **a** $u_5 = 3 \times (-2)^4 = 48$

b $S_{10} = \frac{3[1-(-2)^{10}]}{1-(-2)} = -1023$

c positive terms form GP:
$a = 3,\ r = (-2)^2 = 4$

$S_8 = \frac{3(4^8-1)}{4-1} = 65\,535$

4 GP: $a = 8,\ r = 2,\ n = 10$

$S_{10} = \frac{8(2^{10}-1)}{2-1} = 8184$

6 **a** amount in account after 3rd payment in
$= 200 + (1.005 \times 200) + (1.005^2 \times 200)$
$= 603.005$
interest paid at end of 3rd month
$= 0.005 \times 603.005 = £3.02$ (nearest penny)

b amount paid in $= 12 \times 200 = £2400$

amount in account after 12 months

$= 200(1.005 + 1.005^2 + \ldots + 1.005^{12})$

$= 200 \times S_{12}$ [GP: $a = 1.005,\ r = 1.005$]

$= 200 \times \frac{1.005(1.005^{12}-1)}{1.005-1} = 2479.45$

total interest $= 2479.45 - 2400 = £79.45$

7 $= 1 + 8(-3x) + \frac{8\times7}{2}(-3x)^2$
$+ \frac{8\times7\times6}{3\times2}(-3x)^3 + \ldots$
$= 1 - 24x + 252x^2 - 1512x^3 + \ldots$

8 **a** $S_n = a + ar + ar^2 + \ldots + ar^{n-1}$

$rS_n = ar + ar^2 + \ldots + ar^{n-1} + ar^n$

subtracting, $S_n - rS_n = a - ar^n$
$S_n(1-r) = a(1-r^n)$

$S_n = \frac{a(1-r^n)}{1-r}$

b $r = 6 \div 3 = 2$
$a \times 2^3 = 3 \ \therefore\ a = \frac{3}{8}$

$S_{16} = \frac{\frac{3}{8}(2^{16}-1)}{2-1} = 24\,575\frac{5}{8}$

9 **a** $= 1 + n(ax) + \frac{n(n-1)}{2}(ax)^2 + \ldots$
$= 1 + anx + \frac{1}{2}a^2n(n-1)x^2 + \ldots$

b $\frac{1}{2}a^2n(n-1) = 3an$
$a^2n(n-1) = 6an$
$an[a(n-1) - 6] = 0$
$n \neq 0 \ \therefore\ a(n-1) - 6 = 0$
$an - a = 6$
$n = \frac{6+a}{a}$

c $n = 10 \ \therefore$ coeff. of $x^3 = \frac{10\times9\times8}{3\times2} \times (\frac{2}{3})^3 = 35\frac{5}{9}$

11 **a** $\frac{162}{1-r} = 486$

$1 - r = \frac{162}{486} = \frac{1}{3} \ \therefore\ r = \frac{2}{3}$

b $u_6 = 162 \times (\frac{2}{3})^5 = \frac{64}{3}$ or $21\frac{1}{3}$

c $S_{10} = \frac{162[1-(\frac{2}{3})^{10}]}{1-\frac{2}{3}} = 477.572$

15 **a** 6, 12, 24, 48

b GP: $a = 6,\ r = 2,\ n = 10$

$S_{10} = \frac{6(2^{10}-1)}{2-1} = 6138$

17 **a** $a \times (1.5)^2 = 18$

$a = 18 \div 2.25 = 8$

b $S_6 = \frac{8[(1.5)^6-1]}{1.5-1} = 166.25$

c $8 \times (1.5)^{k-1} > 8000$
$(k-1)\lg 1.5 > \lg 1000$
$k > \frac{\lg 1000}{\lg 1.5} + 1$
$k > 18.04 \ \therefore$ smallest $k = 19$