1 **a** $a = 108,\ ar^3 = 32$

$\therefore\ r^3 = 32 \div 108 = \frac{8}{27}$

$r = \sqrt[3]{\frac{8}{27}} = \frac{2}{3}$

$u_3 = 108 \times (\frac{2}{3})^2 = 48$

b $S_\infty = \dfrac{108}{1-\frac{2}{3}} = 324$

3 **a** new subscribers in 4th week

$= 200 \times (1.15)^3 = 304.175$

$= 304$ (nearest unit)

b new subscribers: GP, $a = 200$, $r = 1.15$

$S_{10} = \dfrac{200[(1.15)^{10} - 1]}{1.15 - 1} = 4060.74$

total no. of subscribers $= 3600 + S_{10}$

$= 7661$ (nearest unit)

6 **a** $r = 3\sqrt{2} \div \sqrt{6} = \sqrt{3}$

$a = \sqrt{6} \div \sqrt{3} = \sqrt{2}$

$x^3 + \dots$ **b** $S_8 = \dfrac{\sqrt{2}[(\sqrt{3})^8 - 1]}{\sqrt{3} - 1}$

$= \dfrac{80\sqrt{2}}{\sqrt{3}-1} \times \dfrac{\sqrt{3}+1}{\sqrt{3}+1}$

$= \dfrac{80\sqrt{2}(\sqrt{3}+1)}{3-1}$

$= 40\sqrt{2}(\sqrt{3}+1)$

7 $\sum_{r=1}^{9} 3^r$: GP, $a = 3$, $r = 3$

$S_9 = \dfrac{3(3^9 - 1)}{3 - 1} = 29523$

$\therefore \sum_{r=1}^{9} (3^r - 1) = 29523 - 9$

$= 29\,514$

17 **a** $S_4 = 3^4 - 1 = 80$

$S_3 = 3^3 - 1 = 26$

$u_4 = S_4 - S_3 = 80 - 26 = 54$

b $S_{n-1} = 3^{n-1} - 1$

$u_n = S_n - S_{n-1}$

$= (3^n - 1) - (3^{n-1} - 1)$

$= 3^n - 3^{n-1}$

$= 3^n(1 - \frac{1}{3}) = \frac{2}{3}(3^n)$ $\quad [k = \frac{2}{3}]$

c $u_{n-1} = \frac{2}{3}(3^{n-1})$

$u_n \div u_{n-1} = \frac{2}{3}(3^n) \div \frac{2}{3}(3^{n-1}) = 3$

$u_n \div u_{n-1}$ is constant $\therefore$ geometric

11 **a** $\dfrac{t}{1-r} = 3t$

$1 - r = \dfrac{t}{3t} = \frac{1}{3} \quad \therefore\ r = \frac{2}{3}$

b $\dfrac{t[1-(\frac{2}{3})^4]}{1-\frac{2}{3}} = 130$

$t = (\frac{1}{3} \times 80) \div \frac{65}{81} = 54$

13 **a** $= 12000 \times (0.75)^4$

$= 3796.875$

$= £3800$ (3sf)

b GP: $a = 12000$, $r = 0.75$

$S_8 = \dfrac{12000[1-(0.75)^8]}{1 - 0.75}$

$= £43\,200$ (3sf)

18 **a** $3(x - 3) = y - 3$

$y = 3x - 6$

b $\left(\dfrac{x}{3}\right)^3 = \dfrac{y}{3}$

$x^3 = 9y = 9(3x - 6)$

$x^3 - 27x + 54 = 0$

c trying $x = 1, 2$ etc. $\Rightarrow$ $x = 3$ is a solution

$\therefore$ $(x - 3)$ is a factor

$$\begin{array}{r|l} & x^2 + 3x - 18 \\ x - 3 & x^3 + 0x^2 - 27x + 54 \\ & x^3 - 3x^2 \\ \hline & 3x^2 - 27x \\ & 3x^2 - 9x \\ \hline & -18x + 54 \\ & -18x + 54 \\ \hline \end{array}$$

$(x - 3)(x^2 + 3x - 18) = 0$

$(x - 3)(x + 6)(x - 3) = 0$

$x = -6$ or 3