

- 1 **a** e^x **b** $3e^x$ **c** $\frac{1}{x}$ **d** $\frac{1}{2x}$
- 2 **a** $-2e^t$ **b** $6t + \frac{1}{t}$ **c** $e^t + 5t^4$ **d** $\frac{3}{2}t^{\frac{1}{2}} + 2e^t$
- e** $\frac{2}{t} + \frac{1}{2}t^{-\frac{1}{2}}$ **f** $2.5e^t - \frac{7}{2t}$ **g** $-t^{-2} + \frac{8}{t}$ **h** $14t - 2 + 4e^t$
- or $\frac{2}{t} + \frac{1}{2\sqrt{t}}$ or $\frac{8}{t} - \frac{1}{t^2}$
- 3 **a** $\frac{dy}{dx} = 12x^2 + e^x$ **b** $\frac{dy}{dx} = 7e^x - 10x + 3$ **c** $\frac{dy}{dx} = \frac{1}{x} + \frac{5}{2}x^{\frac{3}{2}}$
- $\frac{d^2y}{dx^2} = 24x + e^x$ $\frac{d^2y}{dx^2} = 7e^x - 10$ $\frac{d^2y}{dx^2} = -x^{-2} + \frac{15}{4}x^{\frac{1}{2}}$
- d** $\frac{dy}{dx} = 5e^x + \frac{6}{x}$ **e** $\frac{dy}{dx} = -3x^{-2} + \frac{3}{x}$ **f** $\frac{dy}{dx} = 2x^{-\frac{1}{2}} + \frac{1}{4x}$
- $\frac{d^2y}{dx^2} = 5e^x - 6x^{-2}$ $\frac{d^2y}{dx^2} = 6x^{-3} - 3x^{-2}$ $\frac{d^2y}{dx^2} = -x^{-\frac{3}{2}} - \frac{1}{4}x^{-2}$
- 4 **a** $f'(x) = 3 + e^x$ **b** $f'(x) = \frac{1}{x} - 2x$
- $f'(0) = 3 + 1 = 4$ $f'(4) = \frac{1}{4} - 8 = -7\frac{3}{4}$
- c** $f'(x) = \frac{1}{2}x^{-\frac{1}{2}} + \frac{2}{x}$ **d** $f'(x) = 5e^x - 2x^{-3}$
- $f'(9) = \frac{1}{6} + \frac{2}{9} = \frac{7}{18}$ $f'(-\frac{1}{2}) = 5e^{-\frac{1}{2}} + 16$ [19.0 (3sf)]
- 5 **a** $\frac{dy}{dx} = \frac{5}{x} - 8 = 0$ **b** $\frac{dy}{dx} = 2.4e^x - 3.6 = 0$ **c** $\frac{dy}{dx} = 6x - 14 + \frac{4}{x} = 0$
- $5 = 8x$ $e^x = 1.5$ $3x^2 - 7x + 2 = 0$
- $x = \frac{5}{8}$ $x = \ln 1.5$ $(3x - 1)(x - 2) = 0$
- [0.405 (3sf)] $x = \frac{1}{3}, 2$
- 6 **a** $f'(x) = 2e^x - 3 = 7$ **b** $f'(x) = 15 + \frac{1}{x} = 23$
- $e^x = 5$ $\frac{1}{x} = 8$
- $x = \ln 5$ [1.61 (3sf)] $x = \frac{1}{8}$
- c** $f'(x) = \frac{1}{4}x - 2 + \frac{1}{x} = -1$ **d** $f'(x) = \frac{30}{x} - 2x = 4$
- $x^2 - 4x + 4 = 0$ $x^2 + 2x - 15 = 0$
- $(x - 2)^2 = 0$ $(x + 5)(x - 3) = 0$
- $x = 2$ $\ln x$ only real for $x > 0 \therefore x = 3$

$$7 \quad \mathbf{a} \quad \frac{dy}{dx} = e^x - 2$$

$$\text{SP: } e^x - 2 = 0$$

$$x = \ln 2$$

$$\frac{d^2y}{dx^2} = e^x$$

$$x = \ln 2: \frac{d^2y}{dx^2} = 2$$

$$\therefore (\ln 2, 2 - 2 \ln 2), \text{ min}$$

$$\mathbf{b} \quad \frac{dy}{dx} = \frac{1}{x} - 10$$

$$\text{SP: } \frac{1}{x} - 10 = 0$$

$$x = \frac{1}{10}$$

$$\frac{d^2y}{dx^2} = -x^{-2}$$

$$x = \frac{1}{10}: \frac{d^2y}{dx^2} = -100$$

$$\therefore \left(\frac{1}{10}, -1 - \ln 10\right), \text{ max}$$

$$\mathbf{c} \quad \frac{dy}{dx} = \frac{2}{x} - \frac{1}{2}x^{-\frac{1}{2}}$$

$$\text{SP: } \frac{2}{x} - \frac{1}{2}x^{-\frac{1}{2}} = 0$$

$$4 - x^{\frac{1}{2}} = 0$$

$$x^{\frac{1}{2}} = 4, \quad x = 16$$

$$\frac{d^2y}{dx^2} = -2x^{-2} + \frac{1}{4}x^{-\frac{3}{2}}$$

$$x = 16: \frac{d^2y}{dx^2} = -\frac{1}{256}$$

$$\therefore (16, 8 \ln 2 - 4), \text{ max}$$

$$\mathbf{d} \quad \frac{dy}{dx} = 4 - 5e^x$$

$$\text{SP: } 4 - 5e^x = 0$$

$$x = \ln \frac{4}{5}$$

$$\frac{d^2y}{dx^2} = -5e^x$$

$$x = \ln \frac{4}{5}: \frac{d^2y}{dx^2} = -4$$

$$\therefore \left(\ln \frac{4}{5}, 4 \ln \frac{4}{5} - 4\right), \text{ max}$$

$$\mathbf{e} \quad \frac{dy}{dx} = 2 - \frac{4}{x}$$

$$\text{SP: } 2 - \frac{4}{x} = 0$$

$$x = 2$$

$$\frac{d^2y}{dx^2} = 4x^{-2}$$

$$x = 2: \frac{d^2y}{dx^2} = 1$$

$$\therefore (2, 11 - 4 \ln 2), \text{ min}$$

$$\mathbf{f} \quad \frac{dy}{dx} = 2x - 26 + \frac{72}{x}$$

$$\text{SP: } 2x - 26 + \frac{72}{x} = 0$$

$$x^2 - 13x + 36 = 0$$

$$(x - 4)(x - 9) = 0$$

$$x = 4, 9$$

$$\frac{d^2y}{dx^2} = 2 - 72x^{-2}$$

$$x = 4: \frac{d^2y}{dx^2} = -\frac{5}{2}$$

$$x = 9: \frac{d^2y}{dx^2} = \frac{10}{9}$$

$$\therefore (4, 144 \ln 2 - 88), \text{ max}$$

$$(9, 144 \ln 3 - 153), \text{ min}$$

$$8 \quad \frac{dy}{dx} = 1 + ke^x$$

$$\frac{d^2y}{dx^2} = ke^x$$

$$\begin{aligned} \therefore (1-x) \frac{d^2y}{dx^2} + x \frac{dy}{dx} - y &= (1-x)ke^x + x(1+ke^x) - (x+ke^x) \\ &= ke^x - kxe^x + x + kxe^x - x - ke^x = 0 \end{aligned}$$

- 9 a** $x = 2 \therefore y = e^2$
 $\frac{dy}{dx} = e^x$, grad = e^2
 $\therefore y - e^2 = e^2(x - 2)$
 $[y = e^2(x - 1)]$
- b** $x = 3 \therefore y = \ln 3$
 $\frac{dy}{dx} = \frac{1}{x}$, grad = $\frac{1}{3}$
 $\therefore y - \ln 3 = \frac{1}{3}(x - 3)$
 $[y = \frac{1}{3}x + \ln 3 - 1]$
- c** $x = 0 \therefore y = -2$
 $\frac{dy}{dx} = 0.8 - 2e^x$, grad = -1.2
 $\therefore y = -1.2x - 2$
- d** $x = 1 \therefore y = 4$
 $\frac{dy}{dx} = \frac{5}{x} - 4x^{-2}$, grad = 1
 $\therefore y - 4 = x - 1$
 $[y = x + 3]$
- e** $x = 1 \therefore y = 1 - 3e$
 $\frac{dy}{dx} = \frac{1}{3}x^{-\frac{2}{3}} - 3e^x$, grad = $\frac{1}{3} - 3e$
 $\therefore y - (1 - 3e) = (\frac{1}{3} - 3e)(x - 1)$
 $[y = (\frac{1}{3} - 3e)x + \frac{2}{3}]$
- f** $x = 9 \therefore y = \ln 9 - 3$
 $\frac{dy}{dx} = \frac{1}{x} - \frac{1}{2}x^{-\frac{1}{2}}$, grad = $-\frac{1}{18}$
 $\therefore y - (\ln 9 - 3) = -\frac{1}{18}(x - 9)$
 $[y = \ln 9 - \frac{5}{2} - \frac{1}{18}x]$
- 10 a** $x = e \therefore y = 1$
 $\frac{dy}{dx} = \frac{1}{x}$, grad = $\frac{1}{e}$
 \therefore grad of normal = $-e$
 $\therefore y - 1 = -e(x - e)$
 $[y = e^2 + 1 - ex]$
- b** $x = 0 \therefore y = 7$
 $\frac{dy}{dx} = 3e^x$, grad = 3
 \therefore grad of normal = $-\frac{1}{3}$
 $\therefore y - 7 = -\frac{1}{3}x$
- c** $x = 3 \therefore y = 10 + \ln 3$
 $\frac{dy}{dx} = \frac{1}{x}$, grad = $\frac{1}{3}$
 \therefore grad of normal = -3
 $\therefore y - (10 + \ln 3) = -3(x - 3)$
 $[y = 19 + \ln 3 - 3x]$
- d** $x = 1 \therefore y = -2$
 $\frac{dy}{dx} = \frac{3}{x} - 2$, grad = 1
 \therefore grad of normal = -1
 $\therefore y + 2 = -(x - 1)$
 $[y = -x - 1]$
- e** $x = 1 \therefore y = 1$
 $\frac{dy}{dx} = 2x + \frac{8}{x}$, grad = 10
 \therefore grad of normal = $-\frac{1}{10}$
 $\therefore y - 1 = -\frac{1}{10}(x - 1)$
 $[y = \frac{1}{10}(11 - x)]$
- f** $x = 0 \therefore y = -\frac{13}{10}$
 $\frac{dy}{dx} = \frac{1}{10} - \frac{3}{10}e^x$, grad = $-\frac{1}{5}$
 \therefore grad of normal = 5
 $\therefore y = 5x - \frac{13}{10}$