

1 a $x = 0 \therefore y = \frac{1}{10}$
 $\frac{dy}{dx} = \frac{2}{5} + \frac{1}{10}e^x$, grad = $\frac{1}{2}$
 \therefore grad of normal = -2
 $\therefore y = -2x + \frac{1}{10}$
 $20x + 10y - 1 = 0$

b $y = 0 \therefore x = \frac{1}{20}$
 $(\frac{1}{20}, 0)$

3 a $\frac{dy}{dx} = 3 - \frac{1}{2}e^x$
 SP: $3 - \frac{1}{2}e^x = 0$
 $x = \ln 6$
 $\therefore (\ln 6, 3 \ln 6 - 3)$

b $\frac{d^2y}{dx^2} = -\frac{1}{2}e^x$
 $x = \ln 6: \frac{d^2y}{dx^2} = -3$
 \therefore max

5 a $\frac{dy}{dx} = 2 - \frac{1}{x}$, grad = 1
 $\therefore y = x - 1$

b grad of normal = -1
 $\therefore y = -(x - 1)$ [$y = 1 - x$]
 at B, $x = 0 \therefore y = -1$
 at C, $x = 0 \therefore y = 1$
 mid-point of $(0, -1)$ and $(0, 1)$
 $= (0, \frac{-1+1}{2}) = (0, 0)$
 \therefore mid-point of BC is the origin

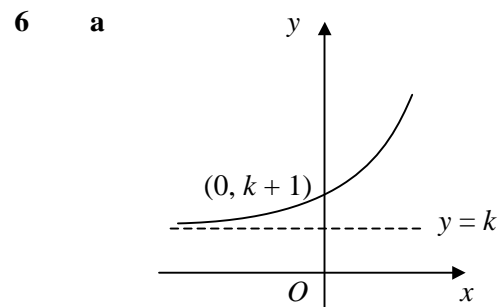
c SP: $2 - \frac{1}{x} = 0$
 $x = \frac{1}{2}$
 $\therefore y = 1 - 2 - \ln \frac{1}{2}$
 $= -1 - \ln 2^{-1}$
 $= \ln 2 - 1$

2 a $x = 1 \therefore y = 5e$
 $\frac{dy}{dx} = 5e^x - \frac{3}{x}$, grad = $5e - 3$
 $\therefore y - 5e = (5e - 3)(x - 1)$
 $y = (5e - 3)x + 3$

b at Q, $x = 0 \therefore y = 3$
 R is $(1, 0)$
 area = $\frac{1}{2} \times (3 + 5e) \times 1$
 $= \frac{1}{2}(5e + 3)$

4 a at P, $x = 4 \therefore y = 6 \ln 4 - 8$
 $\frac{dy}{dx} = \frac{6}{x} - 2x^{-\frac{1}{2}}$, grad = $\frac{1}{2}$
 $\therefore y - (6 \ln 4 - 8) = \frac{1}{2}(x - 4)$
 [$y = \frac{1}{2}x - 10 + 12 \ln 2$]

b at Q, $y = 0 \therefore x = 20 - 24 \ln 2$
 at R, $x = 0 \therefore y = 12 \ln 2 - 10$
 area = $\frac{1}{2} \times (20 - 24 \ln 2) \times (10 - 12 \ln 2)$
 $= (10 - 12 \ln 2)^2$



b $x = 2 \therefore y = e^2 + k$
 $\frac{dy}{dx} = e^x$, grad = e^2
 $\therefore y - (e^2 + k) = e^2(x - 2)$
 [$y = e^2x - e^2 + k$]

c $(-1, 0) \therefore 0 = -e^2 - e^2 + k$
 $k = 2e^2$

7 a $\frac{dy}{dx} = 6x - \frac{2}{x}$

at P , $6x - \frac{2}{x} = -1$

$$6x^2 + x - 2 = 0$$

$$(3x + 2)(2x - 1) = 0$$

$x > 0 \therefore x = \frac{1}{2}$

b $x = 1 \therefore y = 3$, grad = 4

$\therefore y - 3 = 4(x - 1)$

[$y = 4x - 1$]

9 a at P , $x = 0 \therefore y = 3$

$\frac{dy}{dx} = -e^x$, grad = -1

\therefore grad of normal = 1

$\therefore y = x + 3$

b at Q , $y = 0 \therefore x = \ln 4$

grad at $Q = -4$

$\therefore y = -4(x - \ln 4)$ [$y = 8 \ln 2 - 4x$]

c at R $x + 3 = -4(x - \ln 4)$

$$5x = 4 \ln 4 - 3 = 8 \ln 2 - 3$$

$$x = \frac{1}{5}(8 \ln 2 - 3)$$

$\therefore a = \frac{8}{5}$

d $b = -\frac{3}{5}$

8 a $\frac{dy}{dx} = e^x$, grad at $P = e^p$

tangent: $y - e^p = e^p(x - p)$

$(0, 0) \therefore 0 - e^p = e^p(0 - p)$

$$e^p(p - 1) = 0$$

$e^p \neq 0 \therefore p = 1$

b $P(1, e)$, grad at $P = e$

\therefore grad of normal = $-\frac{1}{e}$

$\therefore y - e = -\frac{1}{e}(x - 1)$

at Q , $y = 0 \therefore x = e^2 + 1$

\therefore area = $\frac{1}{2} \times (e^2 + 1) \times e = \frac{1}{2}e(1 + e^2)$

10 $f'(x) = 36x^3 - \frac{16}{x}$

SP: $36x^3 - \frac{16}{x} = 0$

$$x^4 = \frac{4}{9}$$

$$x^2 = -\frac{2}{3} \text{ [no solutions] or } \frac{2}{3}$$

$x > 0 \therefore x = \sqrt{\frac{2}{3}}$

\therefore decreasing for $0 < x \leq \sqrt{\frac{2}{3}}$

$$k = \sqrt{\frac{2}{3}} \text{ or } \frac{1}{3}\sqrt{6}$$