1 **a** $= 5(x+3)^4$ **b** $= 3(2x-1)^2 \times 2 = 6(2x-1)^2$ **c** $= 7(8-x)^6 \times (-1) = -7(8-x)^6$ **d** $= 12(3x+4)^5 \times 3 = 36(3x+4)^5$

e $= 4(6-5x)^3 \times (-5) = -20(6-5x)^3$ **f** $= -(x-2)^{-2}$ **g** $= -12(2x+3)^{-4} \times 2 = -24(2x+3)^{-4}$ **h** $= -2(7-3x)^{-3} \times (-3) = 6(7-3x)^{-3}$

2 **a** $= 6e^{3t}$ **b** $= \frac{1}{2}(4t-1)^{-\frac{1}{2}} \times 4 = 2(4t-1)^{-\frac{1}{2}}$ **c** $= \frac{5}{t}$ **d** $= \frac{3}{2}(8-3t)^{\frac{1}{2}} \times (-3) = -\frac{9}{2}(8-3t)^{\frac{1}{2}}$

e $= \frac{3}{6t+1} \times 6 = \frac{18}{6t+1}$ **f** $= \frac{1}{2}e^{5t+4} \times 5 = \frac{5}{2}e^{5t+4}$ **g** $= \frac{d}{dx}[6(2t-5)^{-\frac{1}{3}}] = -2(2t-5)^{-\frac{4}{3}} \times 2 = -4(2t-5)^{-\frac{4}{3}}$ **h** $= \frac{2}{3-\frac{1}{4}t} \times (-\frac{1}{4}) = \frac{2}{t-12}$

3 **a** $\frac{dy}{dx} = 4(3x-1)^3 \times 3 = 12(3x-1)^3$

$\frac{d^2y}{dx^2} = 36(3x-1)^2 \times 3 = 108(3x-1)^2$

b $\frac{dy}{dx} = \frac{4}{1+2x} \times 2 = 8(1+2x)^{-1}$

$\frac{d^2y}{dx^2} = -8(1+2x)^{-2} \times 2 = \frac{-16}{(1+2x)^2}$

c $\frac{dy}{dx} = \frac{1}{2}(5-2x)^{-\frac{1}{2}} \times (-2) = -(5-2x)^{-\frac{1}{2}}$

$\frac{d^2y}{dx^2} = \frac{1}{2}(5-2x)^{-\frac{3}{2}} \times (-2) = -(5-2x)^{-\frac{3}{2}}$

4 **a** $f'(x) = 2x - \frac{6}{x}$

$f'(3) = 6 - 2 = 4$

b $f'(x) = 2 - e^{x-2}$

$f'(2) = 2 - 1 = 1$

c $f'(x) = 4(2-5x)^3 \times (-5) = -20(2-5x)^3$

$f'(\frac{1}{2}) = -20 \times (-\frac{1}{8}) = \frac{5}{2}$

d $f'(x) = -4(x+5)^{-2}$

$f'(-1) = -4 \times \frac{1}{16} = -\frac{1}{4}$

5 **a** $f'(x) = 2(3x+15)^{-\frac{1}{2}} \times 3 = 2$

$\frac{6}{\sqrt{3x+15}} = 2$

$\sqrt{3x+15} = 3$

$3x + 15 = 9$

$x = -2$

b $f'(x) = 2x - \frac{1}{x-2} = 5$

$2x(x-2) - 1 = 5(x-2)$

$2x^2 - 9x + 9 = 0$

$(2x-3)(x-3) = 0$

for real $f(x)$, $x > 2$ $\therefore x = 3$

6 **a** $= 3(x^2-4)^2 \times 2x = 6x(x^2-4)^2$ **b** $= 12(3x^2+1)^5 \times 6x = 72x(3x^2+1)^5$ **c** $= \frac{1}{3+2x^2} \times 4x = \frac{4x}{3+2x^2}$ **d** $= \frac{d}{dx}[(4-x^2)^3] = 3(4-x^2)^2 \times (-2x) = -6x(4-x^2)^2$

e $= \frac{d}{dx}[(\frac{1}{2}x^4+3)^8] = 8(\frac{1}{2}x^4+3)^7 \times 2x^3 = 16x^3(\frac{1}{2}x^4+3)^7$ **f** $= \frac{d}{dx}[(3-x^2)^{-\frac{1}{2}}] = -\frac{1}{2}(3-x^2)^{-\frac{3}{2}} \times (-2x) = x(3-x^2)^{-\frac{3}{2}}$ **g** $= 7e^{x^2} \times 2x = 14xe^{x^2}$ **h** $= 4(1-5x+x^3)^3 \times (-5+3x^2) = 4(3x^2-5)(1-5x+x^3)^3$

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i $= \frac{3}{4-\sqrt{x}} \times (-\frac{1}{2}x^{-\frac{1}{2}})$ $= \frac{3}{2x-8\sqrt{x}}$

j $= 7(e^{4x}+2)^6 \times 4e^{4x}$ $= 28e^{4x}(e^{4x}+2)^6$

k $= -(5+4\sqrt{x})^{-2} \times 2x^{-\frac{1}{2}}$ $= \frac{-2}{\sqrt{x}(5+4\sqrt{x})^2}$

l $= 5(\frac{2}{x}-x)^4 \times (-2x^{-2}-1)$ $= -5(\frac{2}{x^2}+1)(\frac{2}{x}-x)^4$

7 **a** $\frac{dy}{dx} = 5(2x-3)^4 \times 2$

SP: $10(2x-3)^4 = 0$

$x = \frac{3}{2}$

$\therefore (\frac{3}{2}, 0)$

b $\frac{dy}{dx} = 3(x^2-4)^2 \times 2x$

SP: $6x(x^2-4)^2 = 0$

$x = 0$ or $x^2 = 4$

$x = 0, \pm 2$

$\therefore (-2, 0), (0, -64), (2, 0)$

c $\frac{dy}{dx} = 8 - 2e^{2x}$

SP: $8 - 2e^{2x} = 0$

$e^{2x} = 4$

$x = \frac{1}{2}\ln 4 = \ln 2$

$\therefore (\ln 2, 8\ln 2 - 4)$

d $\frac{dy}{dx} = \frac{1}{2}(1+2x^2)^{-\frac{1}{2}} \times 4x$

SP: $\frac{2x}{\sqrt{1+2x^2}} = 0$

$x = 0$

$\therefore (0, 1)$

e $\frac{dy}{dx} = \frac{2}{x-x^2} \times (1-2x)$

SP: $\frac{2(1-2x)}{x-x^2} = 0$

$x = \frac{1}{2}$

$\therefore (\frac{1}{2}, -4\ln 2)$

f $\frac{dy}{dx} = 4 - (x-3)^{-2}$

SP: $4 - \frac{1}{(x-3)^2} = 0$

$(x-3)^2 = \frac{1}{4}$, $x - 3 = \pm\frac{1}{2}$

$x = \frac{5}{2}, \frac{7}{2}$

$\therefore (\frac{5}{2}, 8), (\frac{7}{2}, 16)$

8 **a** $x = 2 \ \therefore y = 1$

$\frac{dy}{dx} = 4(3x-7)^3 \times 3 = 12(3x-7)^3$

grad $= -12$

$\therefore y - 1 = -12(x-2)$

$[\, y = 25 - 12x \,]$

b $x = 0 \ \therefore y = 2$

$\frac{dy}{dx} = \frac{1}{1+4x} \times 4 = \frac{4}{1+4x}$

grad $= 4$

$\therefore y = 4x + 2$

c $x = 1 \ \therefore y = 3$

$\frac{dy}{dx} = -9(x^2+2)^{-2} \times 2x = -18x(x^2+2)^{-2}$

grad $= -2$

$\therefore y - 3 = -2(x-1)$

$[\, y = 5 - 2x \,]$

d $x = \frac{1}{4} \ \therefore y = \frac{1}{2}$

$\frac{dy}{dx} = \frac{1}{2}(5x-1)^{-\frac{1}{2}} \times 5 = \frac{5}{2}(5x-1)^{-\frac{1}{2}}$

grad $= 5$

$\therefore y - \frac{1}{2} = 5(x - \frac{1}{4})$

$[\, y = 5x - \frac{3}{4} \,]$

9 **a** $x = -2 \ \therefore y = -9$

$\frac{dy}{dx} = e^{4-x^2} \times (-2x) = -2x\,e^{4-x^2}$

grad $= 4 \ \therefore$ grad of normal $= -\frac{1}{4}$

$\therefore y + 9 = -\frac{1}{4}(x+2)$

$[\, y = -\frac{1}{4}x - \frac{19}{2} \,]$

b $x = \frac{1}{2} \ \therefore y = \frac{1}{8}$

$\frac{dy}{dx} = 3(1-2x^2)^2 \times (-4x) = -12x(1-2x^2)^2$

grad $= -\frac{3}{2} \ \therefore$ grad of normal $= \frac{2}{3}$

$\therefore y - \frac{1}{8} = \frac{2}{3}(x - \frac{1}{2})$

$[\, 16x - 24y - 5 = 0 \,]$

c $x = 1 \ \therefore y = \frac{1}{2}$

$\frac{dy}{dx} = -(2-\ln x)^{-2} \times (-\frac{1}{x}) = \frac{1}{x(2-\ln x)^2}$

grad $= \frac{1}{4} \ \therefore$ grad of normal $= -4$

$\therefore y - \frac{1}{2} = -4(x-1)$

$[\, y = \frac{9}{2} - 4x \,]$

d $x = 3 \ \therefore y = 6e$

$\frac{dy}{dx} = 2e^{\frac{x}{3}}$

grad $= 2e \ \therefore$ grad of normal $= -\frac{1}{2e}$

$\therefore y - 6e = -\frac{1}{2e}(x-3)$

$[\, x + 2ey - 12e^2 - 3 = 0 \,]$

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