C3 DIFFERENTIATION — Worksheet E

1 Given that $f(x) = x(x + 2)^3$, find $f'(x)$

a by first expanding $f(x)$, **b** using the product rule.

2 Differentiate each of the following with respect to x and simplify your answers.

a xe^x **b** $x(x + 1)^5$ **c** $x \ln x$ **d** $x^2(x - 1)^3$

e $x^3 \ln 2x$ **f** x^2e^{-x} **g** $2x^4(5 + x)^3$ **h** $x^2(x - 3)^4$

3 Find $\frac{dy}{dx}$, simplifying your answer in each case.

a $y = x(2x - 1)^3$ **b** $y = 3x^4e^{2x + 3}$ **c** $y = x\sqrt{x - 1}$

d $y = x^2 \ln (x + 6)$ **e** $y = x(1 - 5x)^4$ **f** $y = (x + 2)(x - 3)^3$

g $y = x^{\frac{4}{3}}e^{3x}$ **h** $y = (x + 1) \ln (x^2 - 1)$ **i** $y = x^2\sqrt{3x + 1}$

4 Find the value of $f'(x)$ at the value of x indicated in each case.

a $f(x) = 4xe^{3x}$, $x = 0$ **b** $f(x) = 2x(x^2 + 2)^3$, $x = -1$

c $f(x) = (5x - 4) \ln 3x$, $x = \frac{1}{3}$ **d** $f(x) = x^{\frac{1}{2}}(1 - 2x)^3$, $x = \frac{1}{4}$

5 Find the coordinates of any stationary points on each curve.

a $y = xe^{2x}$ **b** $y = x(x - 4)^3$ **c** $y = x^2(2x - 3)^4$

d $y = x\sqrt{x + 12}$ **e** $y = 2 + x^2e^{-4x}$ **f** $y = (1 - 3x)(3 - x)^3$

6 Find an equation for the tangent to each curve at the point on the curve with the given x-coordinate.

a $y = x(x - 2)^4$, $x = 1$ **b** $y = 3x^2e^x$, $x = 1$

c $y = (4x - 1) \ln 2x$, $x = \frac{1}{2}$ **d** $y = x^2\sqrt{x + 6}$, $x = -2$

7 Find an equation for the normal to each curve at the point on the curve with the given x-coordinate. Give your answers in the form $ax + by + c = 0$, where a, b and c are integers.

a $y = x^2(2 - x)^3$, $x = 1$ **b** $y = x \ln (3x - 5)$, $x = 2$

c $y = (x^2 - 1)e^{3x}$, $x = 0$ **d** $y = x\sqrt{x - 4}$, $x = 8$

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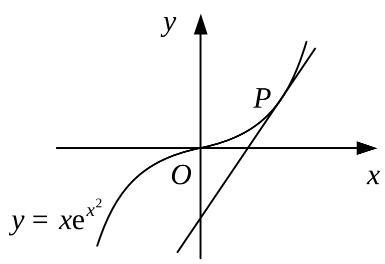

The diagram shows part of the curve with equation $y = xe^{x^2}$ and the tangent to the curve at the point P with x-coordinate 1.

a Find an equation for the tangent to the curve at P.

b Show that the area of the triangle bounded by this tangent and the coordinate axes is $\frac{2}{3}e$.