

- 1 Find an equation for the tangent to the curve with equation

$$y = (3-x)^{\frac{3}{2}}$$

at the point on the curve with  $x$ -coordinate  $-1$ . (4)

- 2 a Sketch the curve with equation  $y = 3 - \ln 2x$ . (2)

b Find the exact coordinates of the point where the curve crosses the  $x$ -axis. (2)

c Find an equation for the tangent to the curve at the point on the curve where  $x = 5$ . (4)

This tangent cuts the  $x$ -axis at  $A$  and the  $y$ -axis at  $B$ .

d Show that the area of triangle  $OAB$ , where  $O$  is the origin, is approximately  $7.20$  (3)

- 3 Differentiate with respect to  $x$

a  $(3x - 1)^4$ , (2)

b  $\frac{x^2}{\sin 2x}$ . (3)

- 4 The area of the surface of a boulder covered by lichen,  $A \text{ cm}^2$ , at time  $t$  years after initial observation, is modelled by the formula

$$A = 2e^{0.5t}.$$

Using this model,

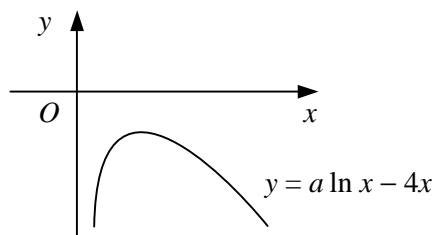
a find the area of lichen on the boulder after three years, (2)

b find the rate at which the area of lichen is increasing per day after three years, (4)

c find, to the nearest year, how long it takes until the area of lichen is  $65 \text{ cm}^2$ . (2)

d Explain why the model cannot be valid for large values of  $t$ . (1)

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The diagram shows the curve with equation  $y = a \ln x - 4x$ , where  $a$  is a positive constant.

Find, in terms of  $a$ ,

a the coordinates of the stationary point on the curve, (4)

b an equation for the tangent to the curve at the point where  $x = 1$ . (3)

Given that this tangent meets the  $x$ -axis at the point  $(3, 0)$ ,

c show that  $a = 6$ . (2)

- 6 Given that  $y = e^{2x} \sin x$ ,

a find  $\frac{dy}{dx}$ , (2)

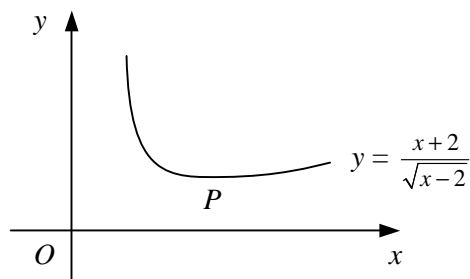
b show that  $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 5y = 0$ . (3)

7 A curve has the equation  $x = \tan^2 y$ .

a Show that  $\frac{dy}{dx} = \frac{1}{2\sqrt{x(x+1)}}$ . (5)

b Find an equation for the normal to the curve at the point where  $y = \frac{\pi}{4}$ . (3)

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The diagram shows the curve  $y = \frac{x+2}{\sqrt{x-2}}$ ,  $x > 2$ , which has a minimum point at  $P$ .

a Find and simplify an expression for  $\frac{dy}{dx}$ . (3)

b Find the coordinates of  $P$ . (2)

The point  $Q$  on the curve has  $x$ -coordinate 3.

c Show that the normal to the curve at  $Q$  has equation

$$2x - 3y + 9 = 0. \quad (3)$$

9 A curve has the equation  $y = e^x(x-1)^2$ .

a Find  $\frac{dy}{dx}$ . (3)

b Show that  $\frac{d^2y}{dx^2} = e^x(x^2 + 2x - 1)$ . (2)

c Find the exact coordinates of the turning points of the curve and determine their nature. (4)

d Show that the tangent to the curve at the point where  $x = 2$  has the equation

$$y = e^2(3x - 5). \quad (3)$$

10 The curve with equation  $y = \frac{1}{2}x^2 - 3 \ln x$ ,  $x > 0$ , has a stationary point at  $A$ .

a Find the exact  $x$ -coordinate of  $A$ . (3)

b Determine the nature of the stationary point. (2)

c Show that the  $y$ -coordinate of  $A$  is  $\frac{3}{2}(1 - \ln 3)$ . (2)

d Find an equation for the tangent to the curve at the point where  $x = 1$ , giving your answer in the form  $ax + by + c = 0$ , where  $a$ ,  $b$  and  $c$  are integers. (3)

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$$f(x) = \frac{6x}{(x-1)(x+2)} - \frac{2}{x-1}.$$

a Show that  $f(x) = \frac{4}{x+2}$ . (5)

b Find an equation for the tangent to the curve  $y = f(x)$  at the point with  $x$ -coordinate 2, giving your answer in the form  $ax + by = c$ , where  $a$ ,  $b$  and  $c$  are integers. (4)