C3 NUMERICAL METHODS Worksheet A

1 Show in each case that there is a root of the equation $f(x) = 0$ in the given interval.

a $f(x) = x^3 + 3x - 7$ $(1, 2)$
b $f(x) = 5\cos x - 3x$ $(0.5, 1)$
c $f(x) = 2e^x + x + 5$ $(-6, -5)$
d $f(x) = x^4 - 5x^2 + 1$ $(2.1, 2.2)$
e $f(x) = \ln(4x - 1) + x^2$ $(0.4, 0.5)$
f $f(x) = e^{-x} - 9\cos 4x$ $(10, 11)$

2 Given that $|N| \leq 5$, find in each case the integer N such that there is a root of the equation $f(x) = 0$ in the interval $(N, N + 1)$.

a $f(x) = x^3 - 3\sqrt{x} - 4$
b $f(x) = x\ln x - \frac{12}{x}$
c $f(x) = 2x^5 + 4x + 15$
d $f(x) = e^{x-1} + 4x - 2$
e $f(x) = e^x - 3\sin x$
f $f(x) = \tan(0.1x) + x - 6$

3 Show in each case that there is a root of the given equation in the given interval.

a $x^3 = 12 - \frac{x}{4}$ $[2, 3]$
b $12e^x = 9 - 4x$ $[-1, 0]$
c $10\ln 3x = 5 - 7x^2$ $[0.47, 0.48]$
d $\sin 4x = 7e^x$ $[-6.5, -6]$
e $4^x = 3x + 10$ $[-4, -3]$
f $\tan(\frac{1}{2}x) = 2x - 1$ $[2.6, 2.7]$

4 In each case there is a root of the equation $f(x) = 0$ in the given interval.
Find the integer, a, such that this root lies in the interval $(\frac{a}{10}, \frac{a+1}{10})$.

a $f(x) = x^4 + \frac{3}{x} - 5$ $(1, 2)$
b $f(x) = x - \ln(6 + x^2)$ $(2, 3)$
c $f(x) = 5x^3 - 3x^2 + 11$ $(-2, -1)$
d $f(x) = \frac{8}{x} - \cos x$ $(11, 12)$
e $f(x) = \operatorname{cosec} x + \sqrt{x}$ $(5, 6)$
f $f(x) = x^2 - 7e^{2x+5}$ $(-3, -2)$

5 **a** On the same set of axes, sketch the graphs of $y = x^3$ and $y = 4 - x$.
b Hence, show that the equation $x^3 + x - 4 = 0$ has exactly one real root.
c Show that this root lies in the interval $(1, 1.5)$.

6 $$f : x \to x\ln x - 1, \ x \in \mathbb{R}, \ x > 0.$$

a On the same set of axes, sketch the curves $y = \ln x$ and $y = \frac{1}{x}$.
b Hence show that the equation $f(x) = 0$ has exactly one real root.

The real root of $f(x) = 0$ is α.

c Find the integer n such that $n < \alpha < n + 1$.

7 **a** On the same set of axes, sketch the curves $y = e^x$ and $y = 5 - x^2$.
b Hence show that the equation $e^x + x^2 - 5 = 0$ has exactly one negative and one positive real root.
c Show that the negative root lies in the interval $(-3, -2)$.

The positive root, α, is such that $\frac{n}{10} < \alpha < \frac{n+1}{10}$, where n is an integer.

d Find the value of n.