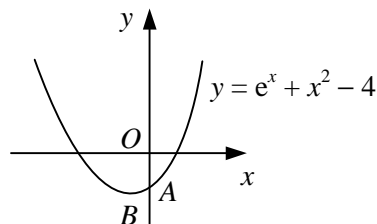


1



The diagram shows the curve $y = e^x + x^2 - 4$. The curve intersects the y -axis at the point A and has a stationary point at B .

a Find $\frac{dy}{dx}$. (1)

b Find an equation for the tangent to the curve at A . (2)

c Show that the x -coordinate of B lies in the interval $[-0.4, -0.3]$. (3)

d Using the iteration formula $x_{n+1} = \frac{1}{3}(x_n - e^{x_n})$, with $x_0 = -0.3$, find the x -coordinate of B correct to 3 decimal places. (4)

2 The function f is defined by

$$f(x) \equiv \sin(x - 6) - \ln(x^2 + 1), \quad x \in \mathbb{R},$$

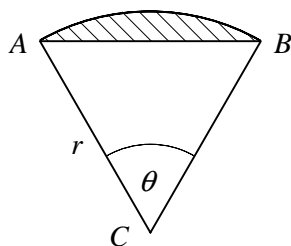
where x is measured in radians.

The equation $f(x) = 0$ has a root in the interval $k < x < k + 1$, where k is a positive integer.

a Find the value of k . (3)

b Use the iteration formula $x_{n+1} = \sqrt{e^{\sin(x_n - 6)} - 1}$, with $x_0 = k$, to find three further approximations for this root, giving your answers to 4 decimal places. (3)

3



The diagram shows a sector ABC of a circle, centre C , radius r . Angle ACB is θ radians.

Given that the ratio of the area of the shaded segment to the area of triangle ABC is $1 : 4$,

a show that $4\theta - 5 \sin \theta = 0$, (4)

b use the iterative formula $\theta_{n+1} = \frac{5}{4} \sin \theta_n$, with $\theta_0 = 1.1$, to find the value of θ correct to 2 decimal places. (4)

4

$$f: x \rightarrow e^{x^2} - x - 3, \quad x \in \mathbb{R}.$$

The equation $f(x) = 0$ can be rearranged into the iterative form $x_{n+1} = \sqrt{\ln(ax_n + b)}$.

a Find the values of the constants a and b in this formula. (3)

The equation $f(x) = 0$ has a solution in the interval $(1, 2)$.

b Using the iterative formula with your values from part **a** and a suitable starting value, find this solution correct to 3 decimal places. (4)

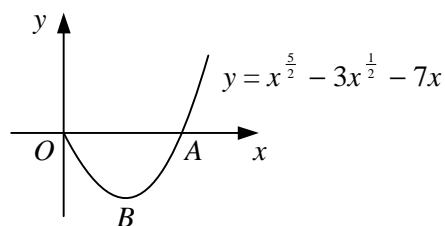
- 5 $f: x \rightarrow x^2 - 9, x \in \mathbb{R}, x \geq 0,$
 $g: x \rightarrow x^3, x \in \mathbb{R}.$
- a Find $f^{-1}(x)$ and state its domain and range. (4)
- b On the same set of axes, sketch the curves $y = f(x)$ and $y = f^{-1}(x)$. (2)
- c Show that the equation $f^{-1}(x) + g(x) = 0$ has a root in the interval $[-2, -1]$. (3)
- d Use the iterative formula $x_{n+1} = -(x_n + 9)^{\frac{1}{6}}$, with $x_0 = -1$, to find this root correct to 3 decimal places. (4)

- 6 a On the same diagram, sketch the curves $y = \frac{1}{x}$ and $y = |-x^2 - 3x|$, showing the coordinates of any points of intersection with the coordinate axes. (3)

The curves intersect at the point P .

- b Show that the x -coordinate of P can be found by solving the equation $x^3 + 3x^2 - 1 = 0$. (3)
- c Use the iteration formula $x_{n+1} = \frac{1}{\sqrt{x_n + 3}}$, with $x_0 = 0$, to find the x -coordinate of P correct to 3 decimal places. (4)

7



The diagram shows the curve $y = x^{\frac{5}{2}} - 3x^{\frac{1}{2}} - 7x, x \geq 0$, which crosses the x -axis at the point A , where $x = \alpha$, and has a stationary point at B , where $x = \beta$.

Show that

- a $4 < \alpha < 5$, (2)
- b $2 < \beta < 3$, (4)
- c $x = \beta$ is a solution to the equation $x = \sqrt{0.6 + 2.8x^{\frac{1}{2}}}$. (3)
- d Use the iterative formula $x_{n+1} = \sqrt{0.6 + 2.8x_n^{\frac{1}{2}}}$, with $x_0 = 2.1$, to find β correct to 4 significant figures. (4)
- 8 The curve with equation $y = 3x - \ln x$ passes through the point $P(1, 3)$.

- a Find an equation for the normal to the curve at P . (4)

The normal to the curve at P intersects the curve again at the point Q .

- b Show that the x -coordinate of Q satisfies the equation

$$2 \ln x - 7x + 7 = 0. \quad (1)$$

The x -coordinate of Q is to be found using an iteration of the form $x_{n+1} = e^{k(x_n-1)}$.

- c Find the value of the constant k . (2)
- d Using $x_0 = 0.5$, find the x -coordinate of Q correct to 3 decimal places. (4)
- e Justify the accuracy of your answer to part d. (2)