

- 1 Find all values of x in the interval $0 \leq x \leq 360^\circ$ for which
- $$\tan^2 x - \sec x = 1. \quad (6)$$
- 2 a Express $2 \cos x^\circ + 5 \sin x^\circ$ in the form $R \cos (x - \alpha)^\circ$, where $R > 0$ and $0 < \alpha < 90$.
Give the values of R and α to 3 significant figures. (4)
- b Solve the equation
- $$2 \cos x^\circ + 5 \cos x^\circ = 3,$$
- for values of x in the interval $0 \leq x \leq 360$, giving your answers to 1 decimal place. (4)
- 3 a Solve the equation
- $$\pi - 6 \arctan 2x = 0,$$
- giving your answer in the form $k\sqrt{3}$. (4)
- b Find the values of x in the interval $0 \leq x \leq 360^\circ$ for which
- $$2 \sin 2x = 3 \cos x,$$
- giving your answers to an appropriate degree of accuracy. (6)
- 4 a Use the identities for $\sin (A + B)$ and $\sin (A - B)$ to prove that
- $$\sin P - \sin Q \equiv 2 \cos \frac{P+Q}{2} \sin \frac{P-Q}{2}. \quad (4)$$
- b Hence, or otherwise, find the values of x in the interval $0 \leq x \leq 180^\circ$ for which
- $$\sin 4x = \sin 2x. \quad (6)$$
- 5 a Prove the identity
- $$(2 \sin \theta - \operatorname{cosec} \theta)^2 \equiv \operatorname{cosec}^2 \theta - 4 \cos^2 \theta, \quad \theta \neq n\pi, \quad n \in \mathbb{Z}. \quad (3)$$
- b i Sketch the curve $y = 3 + 2 \sec x$ for x in the interval $0 \leq x \leq 2\pi$.
ii Write down the coordinates of the point where the curve meets the y -axis.
iii Find the coordinates of the points where the curve crosses the x -axis in this interval. (7)
- 6 a Find the exact values of R and α , where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$, for which
- $$\cos x - \sin x \equiv R \cos (x + \alpha). \quad (3)$$
- b Using the identity
- $$\cos X + \cos Y \equiv 2 \cos \frac{X+Y}{2} \cos \frac{X-Y}{2},$$
- or otherwise, find in terms of π the values of x in the interval $[0, 2\pi]$ for which
- $$\cos x + \sqrt{2} \cos \left(3x - \frac{\pi}{4}\right) = \sin x. \quad (7)$$
- 7 a Prove the identity
- $$\cot 2x + \operatorname{cosec} 2x \equiv \cot x, \quad x \neq \frac{n\pi}{2}, \quad n \in \mathbb{Z}. \quad (4)$$
- b Hence, for x in the interval $0 \leq x \leq 2\pi$, solve the equation
- $$\cot 2x + \operatorname{cosec} 2x = 6 - \cot^2 x,$$
- giving your answers correct to 2 decimal places. (6)

- 8 a Prove that for all real values of x

$$\cos(x + 30)^\circ + \sin x^\circ \equiv \cos(x - 30)^\circ. \quad (4)$$

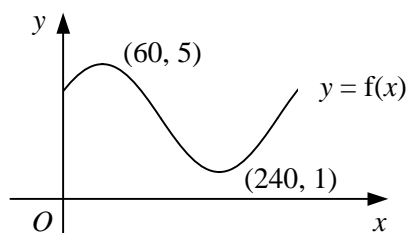
- b Hence, find the exact value of $\cos 75^\circ - \cos 15^\circ$, giving your answer in the form $k\sqrt{2}$. (3)

- c Solve the equation

$$3 \cos(x + 30)^\circ + \sin x^\circ = 3 \cos(x - 30)^\circ + 1$$

$$\text{for } x \text{ in the interval } -180 \leq x \leq 180. \quad (4)$$

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The diagram shows the curve $y = f(x)$ where

$$f(x) \equiv a + b \sin x^\circ + c \cos x^\circ, \quad x \in \mathbb{R}, \quad 0 \leq x \leq 360,$$

The curve has turning points with coordinates (60, 5) and (240, 1) as shown.

- a State, with a reason, the value of the constant a . (2)

- b Find the values of k and α , where $k > 0$ and $0 < \alpha < 90$, such that

$$f(x) = a + k \sin(x + \alpha)^\circ. \quad (3)$$

- c Hence, or otherwise, find the exact values of the constants b and c . (3)

- 10 a Prove the identity

$$\frac{1 - \cos x}{1 + \cos x} \equiv \tan^2 \frac{x}{2}, \quad x \neq (2n + 1)\pi, \quad n \in \mathbb{Z}. \quad (4)$$

- b Use the identity in part a to

- i find the value of $\tan^2 \frac{\pi}{12}$ in the form $a + b\sqrt{3}$, where a and b are integers,

- ii solve the equation

$$\frac{1 - \cos x}{1 + \cos x} = 1 - \sec \frac{x}{2},$$

$$\text{for } x \text{ in the interval } 0 \leq x \leq 2\pi, \text{ giving your answers in terms of } \pi. \quad (9)$$

- 11 a Prove that there are no real values of x for which

$$6 \cot^2 x - \operatorname{cosec} x + 5 = 0. \quad (4)$$

- b Find the values of y in the interval $0 \leq y \leq 180^\circ$ for which

$$\cos 5y = \cos y. \quad (6)$$

- 12 a Use the identities for $\cos(A + B)$ and $\cos(A - B)$ to prove that

$$\sin A \sin B \equiv \frac{1}{2} [\cos(A - B) - \cos(A + B)]. \quad (2)$$

- b Hence, or otherwise, find the values of x in the interval $0 \leq x \leq \pi$ for which

$$4 \sin\left(x + \frac{\pi}{3}\right) = \operatorname{cosec}\left(x - \frac{\pi}{6}\right),$$

$$\text{giving your answers as exact multiples of } \pi. \quad (7)$$