1 **a** $\frac{dx}{dt} = 1, \quad \frac{dy}{dt} = 2t$

b $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = 2t \div 1 = 2t$

2 **a** $\frac{dx}{dt} = 2t, \quad \frac{dy}{dt} = 3$

$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{3}{2t}$

b $\frac{dx}{dt} = 2t, \quad \frac{dy}{dt} = 6t^2 + 2t$

$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{6t^2 + 2t}{2t} = 3t + 1$

c $\frac{dx}{dt} = 2\cos t, \quad \frac{dy}{dt} = -6\sin t$

$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{-6\sin t}{2\cos t} = -3\tan t$

d $\frac{dx}{dt} = 3, \quad \frac{dy}{dt} = t^{-2}$

$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{t^{-2}}{3}$

$= \frac{1}{3t^2}$

e $\frac{dx}{dt} = -2\sin 2t, \quad \frac{dy}{dt} = \cos t$

$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{\cos t}{-2\sin 2t}$

$= \frac{\cos t}{-4\sin t\cos t} = -\frac{1}{4}\operatorname{cosec} t$

f $\frac{dx}{dt} = e^{t+1}, \quad \frac{dy}{dt} = 2e^{2t-1}$

$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{2e^{2t-1}}{e^{t+1}}$

$= 2e^{t-2}$

g $\frac{dx}{dt} = 2\sin t \times \cos t,$

$\frac{dy}{dt} = 3\cos^2 t \times (-\sin t)$

$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{-3\cos^2 t\sin t}{2\sin t\cos t}$

$= -\frac{3}{2}\cos t$

h $\frac{dx}{dt} = 3\sec t\tan t,$

$\frac{dy}{dt} = 5\sec^2 t$

$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{5\sec^2 t}{3\sec t\tan t}$

$= \frac{5\sec t}{3\tan t} = \frac{5}{3}\operatorname{cosec} t$

i $\frac{dx}{dt} = -(t+1)^{-2},$

$\frac{dy}{dt} = \frac{1\times(t-1) - t\times 1}{(t-1)^2} = -(t-1)^{-2}$

$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{-(t-1)^{-2}}{-(t+1)^{-2}}$

$= \frac{(t+1)^2}{(t-1)^2} = \left(\frac{t+1}{t-1}\right)^2$

3 **a** $t = 1 \quad \therefore x = 1, \; y = 3$

$\frac{dx}{dt} = 3t^2, \quad \frac{dy}{dt} = 6t$

$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{6t}{3t^2} = \frac{2}{t}$

grad = 2

$\therefore \; y - 3 = 2(x - 1)$

$y = 2x + 1$

b $t = 2 \quad \therefore x = -3, \; y = 0$

$\frac{dx}{dt} = -2t, \quad \frac{dy}{dt} = 2 - 2t$

$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{2-2t}{-2t} = 1 - \frac{1}{t}$

grad = $\frac{1}{2}$

$\therefore \; y - 0 = \frac{1}{2}(x + 3)$

$y = \frac{1}{2}x + \frac{3}{2}$

c $t = \frac{\pi}{3} \quad \therefore x = \sqrt{3}, \; y = -1$

$\frac{dx}{dt} = 2\cos t, \quad \frac{dy}{dt} = 4\sin t$

$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{4\sin t}{2\cos t} = 2\tan t$

grad = $2\sqrt{3}$

$\therefore \; y + 1 = 2\sqrt{3}(x - \sqrt{3})$

$y = 2\sqrt{3}\,x - 7$

d $t = 3 \quad \therefore x = 0, \; y = 4$

$\frac{dx}{dt} = -\frac{1}{4-t} = \frac{1}{t-4}, \quad \frac{dy}{dt} = 2t$

$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{2t}{\frac{1}{t-4}} = 2t(t-4)$

grad = -6

$\therefore \; y - 4 = -6(x - 0)$

$y = 4 - 6x$

4 $\theta = \frac{\pi}{3} \quad \therefore\ x = 2,\ y = 2\sqrt{3}$

$\frac{dx}{d\theta} = \sec\theta\tan\theta,\ \frac{dy}{d\theta} = 2\sec^2\theta$

$\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta} = \frac{2\sec^2\theta}{\sec\theta\tan\theta}$

$= \frac{2\sec\theta}{\tan\theta} = 2\,\text{cosec}\,\theta$

grad $= \frac{4}{\sqrt{3}} \quad \therefore$ grad of normal $= -\frac{\sqrt{3}}{4}$

$\therefore\ y - 2\sqrt{3} = -\frac{\sqrt{3}}{4}(x - 2)$

$4y - 8\sqrt{3} = -\sqrt{3}\,x + 2\sqrt{3}$

$\sqrt{3}\,x + 4y = 10\sqrt{3}$

5 **a** $\frac{dx}{dt} = -t^{-2},\ \frac{dy}{dt} = -(t + 2)^{-2}$

$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{-(t+2)^{-2}}{-t^{-2}}$

$= \frac{t^2}{(t+2)^2} = \left(\frac{t}{t+2}\right)^2$

b $t = 2 \quad \therefore\ x = \frac{1}{2},\ y = \frac{1}{4}$

grad $= \frac{1}{4} \quad \therefore$ grad of normal $= -4$

$\therefore\ y - \frac{1}{4} = -4(x - \frac{1}{2})$

$4y - 1 = -16x + 8$

$16x + 4y - 9 = 0$

6 **a** $\frac{dx}{dt} = 2\cos 2t$

$\frac{dy}{dt} = 2\sin t \times \cos t = \sin 2t$

$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{\sin 2t}{2\cos 2t} = \frac{1}{2}\tan 2t$

b $t = \frac{\pi}{6} \quad \therefore\ x = \frac{\sqrt{3}}{2},\ y = \frac{1}{4}$

grad $= \frac{\sqrt{3}}{2}$

$\therefore\ y - \frac{1}{4} = \frac{\sqrt{3}}{2}(x - \frac{\sqrt{3}}{2})$

$[\ \sqrt{3}\,x - 2y - 1 = 0\ ]$

7 **a** $\frac{dx}{d\theta} = -3\sin\theta,\ \frac{dy}{d\theta} = 4\cos\theta$

$\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta} = -\frac{4\cos\theta}{3\sin\theta}$

at $(3\cos\alpha, 4\sin\alpha)$, $\theta = \alpha$

$\therefore$ grad $= -\frac{4\cos\alpha}{3\sin\alpha}$

$\therefore\ y - 4\sin\alpha = -\frac{4\cos\alpha}{3\sin\alpha}(x - 3\cos\alpha)$

$3y\sin\alpha - 12\sin^2\alpha = -4x\cos\alpha + 12\cos^2\alpha$

$3y\sin\alpha + 4x\cos\alpha = 12(\cos^2\alpha + \sin^2\alpha)$

$3y\sin\alpha + 4x\cos\alpha = 12$

b at $(-\frac{3}{2},\ 2\sqrt{3})$,

$3\cos\alpha = -\frac{3}{2} \ \Rightarrow\ \cos\alpha = -\frac{1}{2}$

$4\sin\alpha = 2\sqrt{3} \ \Rightarrow\ \sin\alpha = \frac{\sqrt{3}}{2}$

$\therefore\ 3y \times \frac{\sqrt{3}}{2} + 4x \times (-\frac{1}{2}) = 12$

$4x - 3\sqrt{3}\,y + 24 = 0$

8 **a** $x = 0 \ \Rightarrow\ t^2 = 0 \qquad \Rightarrow\ t = 0$

$y = 0 \ \Rightarrow\ t(t - 2) = 0 \ \Rightarrow\ t = 0, 2$

$\therefore$ (0, 0), (4, 0)

b **i** $\frac{dx}{dt} = 2t,\ \frac{dy}{dt} = 2t - 2$

$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{2t-2}{2t} = 1 - t^{-1}$

$t = x^{\frac{1}{2}} \quad \therefore\ \frac{dy}{dx} = 1 - x^{-\frac{1}{2}}$

ii $y = x^{\frac{1}{2}}(x^{\frac{1}{2}} - 2) = x - 2x^{\frac{1}{2}}$

$\frac{dy}{dx} = 1 - x^{-\frac{1}{2}}$

9 **a** $\frac{dx}{d\theta} = 2\sin\theta,\ \frac{dy}{d\theta} = 3\cos\theta$

$\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta} = \frac{3\cos\theta}{2\sin\theta}$ or $\frac{3}{2}\cot\theta$

b **i** $\frac{3\cos\theta}{2\sin\theta} = 0 \ \therefore\ \cos\theta = 0$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2} \quad \therefore$ (1, 3), (1, –3)

ii $\frac{3\cos\theta}{2\sin\theta} \to \infty \ \therefore\ \sin\theta = 0$

$\theta = 0, \pi \ \therefore$ (–1, 0), (3, 0)

10 **a** $x = 0 \Rightarrow \sin\theta = 0 \Rightarrow \theta = 0$

$y = 0 \Rightarrow \sin 2\theta = 0 \Rightarrow \theta = 0, \frac{\pi}{2}$

$\therefore$ (0, 0), (1, 0)

b $\frac{dx}{d\theta} = \cos\theta$, $\frac{dy}{d\theta} = 2\cos 2\theta$

$\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta} = \frac{2\cos 2\theta}{\cos\theta}$

$\frac{2\cos 2\theta}{\cos\theta} = 0 \quad \therefore \cos 2\theta = 0$

$\theta = \frac{\pi}{4} \quad \therefore y = 1$

c $y = \sin 2\theta = 2\sin\theta\cos\theta$

$\cos\theta = \pm\sqrt{1 - \sin^2\theta}$

$0 \le \theta \le \frac{\pi}{2} \quad \therefore \cos\theta = \sqrt{1 - \sin^2\theta}$

$\therefore y = 2x\sqrt{1 - x^2}$

11 **a** $t = \frac{\pi}{4}$, $x = \frac{1}{2}$, $y = 1$

$\frac{dx}{dt} = 2\sin t\cos t$, $\frac{dy}{dt} = \sec^2 t$

$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{\sec^2 t}{2\sin t\cos t} = \frac{1}{2}\sec^3 t \operatorname{cosec} t$

grad = $\frac{1}{2} \times (\sqrt{2})^3 \times \sqrt{2} = 2$

$\therefore y - 1 = 2(x - \frac{1}{2})$

$y = 2x$

when $x = 0$, $y = 0$

$\therefore$ passes through origin

b $y^2 = \tan^2 t = \frac{\sin^2 t}{\cos^2 t} = \frac{\sin^2 t}{1 - \sin^2 t}$

$\therefore y^2 = \frac{x}{1 - x}$

12 **a** $t = 3$, $x = \frac{10}{3}$, $y = \frac{8}{3}$

$\frac{dx}{dt} = 1 - t^{-2}$, $\frac{dy}{dt} = 1 + t^{-2}$

$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{1 + t^{-2}}{1 - t^{-2}} = \frac{t^2 + 1}{t^2 - 1}$

grad = $\frac{5}{4}$

$\therefore y - \frac{8}{3} = \frac{5}{4}(x - \frac{10}{3})$

[$5x - 4y - 6 = 0$]

b sub. parametric eqns into tangent

$5(t + \frac{1}{t}) - 4(t - \frac{1}{t}) - 6 = 0$

$5(t^2 + 1) - 4(t^2 - 1) - 6t = 0$

$t^2 - 6t + 9 = 0$

$(t - 3)^2 = 0$

$\therefore t = 3$ (at P), no other solutions

$\therefore$ does not intersect again

c $x^2 = (t + \frac{1}{t})^2 = t^2 + 2 + \frac{1}{t^2}$ (1)

$y^2 = (t - \frac{1}{t})^2 = t^2 - 2 + \frac{1}{t^2}$ (2)

(1) – (2) $\Rightarrow$ $x^2 - y^2 = 4$ [$k = 4$]