**1** A curve has the equation

$$3x^2 + xy - y^2 + 9 = 0.$$

Find an expression for $\frac{dy}{dx}$ in terms of $x$ and $y$. **(5)**

**2** A curve has parametric equations

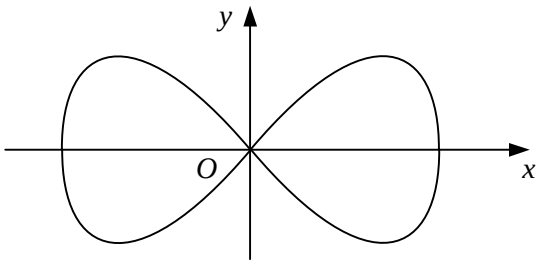
$$x = a\cos\theta, \quad y = a(\sin\theta - \theta), \quad 0 \leq \theta < \pi,$$

where $a$ is a positive constant.

**a** Show that $\frac{dy}{dx} = \tan\frac{\theta}{2}$. **(5)**

**b** Find, in terms of $a$, an equation for the tangent to the curve at the point where it crosses the $y$-axis. **(3)**

**3**




The diagram shows the curve with parametric equations

$$x = \cos\theta, \quad y = \tfrac{1}{2}\sin 2\theta, \quad 0 \leq \theta < 2\pi.$$

**a** Find $\frac{dy}{dx}$ in terms of $\theta$. **(3)**

**b** Find the two values of $\theta$ for which the curve passes through the origin. **(2)**

**c** Show that the two tangents to the curve at the origin are perpendicular to each other. **(2)**

**d** Find a cartesian equation for the curve. **(4)**

**4** A curve has the equation

$$x^2 - 4xy + y^2 = 24.$$

**a** Show that $\frac{dy}{dx} = \frac{x - 2y}{2x - y}$. **(4)**

**b** Find an equation for the tangent to the curve at the point $P$ (2, 10). **(3)**

The tangent to the curve at $Q$ is parallel to the tangent at $P$.

**c** Find the coordinates of $Q$. **(4)**

**5** A curve is given by the parametric equations

$$x = t^2 + 2, \quad y = t(t - 1).$$

**a** Find the coordinates of any points on the curve where the tangent to the curve is parallel to the $x$-axis. **(5)**

**b** Show that the tangent to the curve at the point (3, 2) has the equation

$$3x - 2y = 5.$$ **(5)**

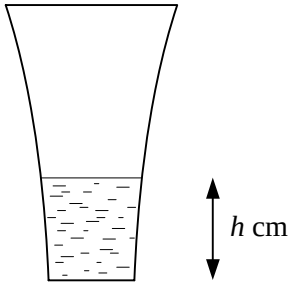
**6** Find an equation for the normal to the curve with equation

$$x^3 - 3x + xy - 2y^2 + 3 = 0$$

at the point (1, 1).

Give your answer in the form $y = mx + c$. **(7)**

**7**




The diagram shows the cross-section of a vase. The volume of water in the vase, $V$ cm$^3$, when the depth of water in the vase is $h$ cm is given by

$$V = 40\pi(e^{0.1h} - 1).$$

The vase is initially empty and water is poured into it at a constant rate of 80 cm$^3$ s$^{-1}$.

Find the rate at which the depth of water in the vase is increasing

**a** when $h = 4$, **(5)**

**b** after 5 seconds of pouring water in. **(4)**

**8** A curve is given by the parametric equations

$$x = \frac{t}{1+t}, \quad y = \frac{t}{1-t}, \quad t \neq \pm 1.$$

**a** Show that $\dfrac{dy}{dx} = \left(\dfrac{1+t}{1-t}\right)^2$. **(4)**

**b** Show that the normal to the curve at the point $P$, where $t = \frac{1}{2}$, has the equation

$$3x + 27y = 28.$$ **(4)**

The normal to the curve at $P$ meets the curve again at the point $Q$.

**c** Find the exact value of the parameter $t$ at $Q$. **(4)**

**9** A curve has the equation

$$2x + x^2y - y^2 = 0.$$

Find the coordinates of the point on the curve where the tangent is parallel to the $x$-axis. **(8)**

**10** A curve has parametric equations

$$x = a\sec\theta, \quad y = 2a\tan\theta, \quad -\tfrac{\pi}{2} \leq \theta < \tfrac{\pi}{2},$$

where $a$ is a positive constant.

**a** Find $\dfrac{dy}{dx}$ in terms of $\theta$. **(3)**

**b** Show that the normal to the curve at the point where $\theta = \frac{\pi}{4}$ has the equation

$$x + 2\sqrt{2}\,y = 5\sqrt{2}\,a.$$ **(4)**

**c** Find a cartesian equation for the curve in the form $y^2 = f(x)$. **(3)**