

$$1 \quad u = x, \frac{du}{dx} = 1; \frac{dv}{dx} = \cos x, v = \sin x$$

$$\int x \cos x \, dx = x \sin x - \int \sin x \, dx \\ = x \sin x + \cos x + c$$

$$2 \quad \mathbf{a} \quad u = x, \frac{du}{dx} = 1; \frac{dv}{dx} = e^x, v = e^x$$

$$\int x e^x \, dx = x e^x - \int e^x \, dx \\ = x e^x - e^x + c \\ = e^x(x - 1) + c$$

$$\mathbf{b} \quad u = 4x, \frac{du}{dx} = 4; \frac{dv}{dx} = \sin x, v = -\cos x$$

$$\int 4x \sin x \, dx = -4x \cos x - \int -4 \cos x \, dx \\ = -4x \cos x + \int 4 \cos x \, dx \\ = -4x \cos x + 4 \sin x + c$$

$$\mathbf{c} \quad u = x, \frac{du}{dx} = 1; \frac{dv}{dx} = \cos 2x, v = \frac{1}{2} \sin 2x$$

$$\int x \cos 2x \, dx = \frac{1}{2} x \sin 2x - \int \frac{1}{2} \sin 2x \, dx \\ = \frac{1}{2} x \sin 2x + \frac{1}{4} \cos 2x + c$$

$$\mathbf{d} \quad u = x, \frac{du}{dx} = 1; \frac{dv}{dx} = (x+1)^{\frac{1}{2}}, v = \frac{2}{3}(x+1)^{\frac{3}{2}}$$

$$\int x \sqrt{x+1} \, dx = \frac{2}{3} x(x+1)^{\frac{3}{2}} - \int \frac{2}{3}(x+1)^{\frac{3}{2}} \, dx \\ = \frac{2}{3} x(x+1)^{\frac{3}{2}} - \frac{4}{15}(x+1)^{\frac{5}{2}} + c \\ = \frac{2}{15}(x+1)^{\frac{3}{2}} [5x - 2(x+1)] + c \\ = \frac{2}{15}(3x-2)(x+1)^{\frac{3}{2}} + c$$

$$\mathbf{e} \quad u = x, \frac{du}{dx} = 1; \frac{dv}{dx} = e^{-3x}, v = -\frac{1}{3}e^{-3x}$$

$$\int \frac{x}{e^{3x}} \, dx = -\frac{1}{3} x e^{-3x} - \int -\frac{1}{3} e^{-3x} \, dx \\ = -\frac{1}{3} x e^{-3x} + \int \frac{1}{3} e^{-3x} \, dx \\ = -\frac{1}{3} x e^{-3x} - \frac{1}{9} e^{-3x} + c \\ = -\frac{1}{9} e^{-3x}(3x+1) + c$$

$$\mathbf{f} \quad u = x, \frac{du}{dx} = 1; \frac{dv}{dx} = \sec^2 x, v = \tan x$$

$$\int x \sec^2 x \, dx = x \tan x - \int \tan x \, dx \\ = x \tan x + \int \frac{-\sin x}{\cos x} \, dx \\ = x \tan x + \ln |\cos x| + c$$

$$3 \quad \mathbf{i} \quad u = x, \frac{du}{dx} = 1; \frac{dv}{dx} = (2x+1)^3, v = \frac{1}{8}(2x+1)^4$$

$$\int x(2x+1)^3 \, dx = \frac{1}{8} x(2x+1)^4 - \int \frac{1}{8}(2x+1)^4 \, dx \\ = \frac{1}{8} x(2x+1)^4 - \frac{1}{80}(2x+1)^5 + c \\ = \frac{1}{80}(2x+1)^4 [10x - (2x+1)] + c \\ = \frac{1}{80}(8x-1)(2x+1)^4 + c$$

$$\mathbf{ii} \quad u = 2x+1 \quad \therefore x = \frac{1}{2}(u-1), \frac{du}{dx} = 2$$

$$\int x(2x+1)^3 \, dx = \int \frac{1}{2}(u-1)u^3 \times \frac{1}{2} \, du \\ = \frac{1}{4} \int (u^4 - u^3) \, du \\ = \frac{1}{4} \left(\frac{1}{5} u^5 - \frac{1}{4} u^4 \right) + c \\ = \frac{1}{4} \left[\frac{1}{5} (2x+1)^5 - \frac{1}{4} (2x+1)^4 \right] + c \\ = \frac{1}{80} (2x+1)^4 [4(2x+1) - 5] + c \\ = \frac{1}{80} (8x-1)(2x+1)^4 + c, \text{ as for part i}$$

$$4 \quad u = x, \frac{du}{dx} = 1; \frac{dv}{dx} = e^{-x}, v = -e^{-x}$$

$$\begin{aligned} \int_0^2 x e^{-x} dx &= [-x e^{-x}]_0^2 - \int_0^2 -e^{-x} dx = [-x e^{-x}]_0^2 + \int_0^2 e^{-x} dx \\ &= [-x e^{-x} - e^{-x}]_0^2 = (-2e^{-2} - e^{-2}) - (0 - 1) \\ &= 1 - 3e^{-2} \end{aligned}$$

$$5 \quad \mathbf{a} \quad u = x, \frac{du}{dx} = 1; \frac{dv}{dx} = \cos x, v = \sin x$$

$$\begin{aligned} \int_0^{\frac{\pi}{6}} x \cos x dx &= [x \sin x]_0^{\frac{\pi}{6}} - \int_0^{\frac{\pi}{6}} \sin x dx \\ &= [x \sin x + \cos x]_0^{\frac{\pi}{6}} \\ &= \left(\frac{\pi}{12} + \frac{\sqrt{3}}{2}\right) - (0 + 1) \\ &= \frac{1}{12}(\pi + 6\sqrt{3} - 12) \end{aligned}$$

$$\mathbf{b} \quad u = x, \frac{du}{dx} = 1; \frac{dv}{dx} = e^{2x}, v = \frac{1}{2}e^{2x}$$

$$\begin{aligned} \int_0^1 x e^{2x} dx &= \left[\frac{1}{2}x e^{2x}\right]_0^1 - \int_0^1 \frac{1}{2}e^{2x} dx \\ &= \left[\frac{1}{2}x e^{2x} - \frac{1}{4}e^{2x}\right]_0^1 \\ &= \left(\frac{1}{2}e^2 - \frac{1}{4}e^2\right) - (0 - \frac{1}{4}) \\ &= \frac{1}{4}(e^2 + 1) \end{aligned}$$

$$\mathbf{c} \quad u = x, \frac{du}{dx} = 1; \frac{dv}{dx} = \sin 3x, v = -\frac{1}{3}\cos 3x$$

$$\begin{aligned} \int_0^{\frac{\pi}{4}} x \sin 3x dx &= \left[-\frac{1}{3}x \cos 3x\right]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} -\frac{1}{3}\cos 3x dx \\ &= \left[-\frac{1}{3}x \cos 3x\right]_0^{\frac{\pi}{4}} + \int_0^{\frac{\pi}{4}} \frac{1}{3}\cos 3x dx \\ &= \left[-\frac{1}{3}x \cos 3x + \frac{1}{9}\sin 3x\right]_0^{\frac{\pi}{4}} \\ &= \left[-\frac{\pi}{12}\left(-\frac{1}{\sqrt{2}}\right) + \frac{1}{9}\left(\frac{1}{\sqrt{2}}\right)\right] - (0) \\ &= \frac{1}{72}\sqrt{2}(3\pi + 4) \end{aligned}$$

$$6 \quad \mathbf{a} \quad u = x^2, \frac{du}{dx} = 2x; \frac{dv}{dx} = e^x, v = e^x$$

$$\begin{aligned} \int x^2 e^x dx &= x^2 e^x - \int 2x e^x dx \\ \text{for } \int 2x e^x dx, \quad u &= 2x, \frac{du}{dx} = 2; \frac{dv}{dx} = e^x, v = e^x \\ \int 2x e^x dx &= 2x e^x - \int 2e^x dx \\ &= 2x e^x - 2e^x + c \\ \therefore \int x^2 e^x dx &= x^2 e^x - (2x e^x - 2e^x) + c \\ &= e^x(x^2 - 2x + 2) + c \end{aligned}$$

$$\mathbf{b} \quad u = e^x, \frac{du}{dx} = e^x; \frac{dv}{dx} = \sin x, v = -\cos x$$

$$\begin{aligned} \int e^x \sin x dx &= -e^x \cos x - \int -e^x \cos x dx \\ &= -e^x \cos x + \int e^x \cos x dx \\ \text{for } \int e^x \cos x dx, \quad u &= e^x, \frac{du}{dx} = e^x; \frac{dv}{dx} = \cos x, v = \sin x \\ \int e^x \cos x dx &= e^x \sin x - \int e^x \sin x dx \\ \therefore \int e^x \sin x dx &= -e^x \cos x + e^x \sin x - \int e^x \sin x dx \\ 2 \int e^x \sin x dx &= -e^x \cos x + e^x \sin x + c \\ \int e^x \sin x dx &= \frac{1}{2}e^x(\sin x - \cos x) + c \end{aligned}$$

7 a $u = x^2$, $\frac{du}{dx} = 2x$; $\frac{dv}{dx} = \sin x$, $v = -\cos x$

$$\begin{aligned}\int x^2 \sin x \, dx &= -x^2 \cos x - \int -2x \cos x \, dx \\ &= -x^2 \cos x + \int 2x \cos x \, dx\end{aligned}$$

for $\int 2x \cos x \, dx$, $u = 2x$, $\frac{du}{dx} = 2$; $\frac{dv}{dx} = \cos x$, $v = \sin x$

$$\begin{aligned}\int 2x \cos x \, dx &= 2x \sin x - \int 2 \sin x \, dx \\ &= 2x \sin x + 2 \cos x + c\end{aligned}$$

$$\begin{aligned}\therefore \int x^2 \sin x \, dx &= -x^2 \cos x + (2x \sin x + 2 \cos x) + c \\ &= (2 - x^2)\cos x + 2x \sin x + c\end{aligned}$$

b $u = x^2$, $\frac{du}{dx} = 2x$; $\frac{dv}{dx} = e^{3x}$, $v = \frac{1}{3}e^{3x}$

$$\int x^2 e^{3x} \, dx = \frac{1}{3}x^2 e^{3x} - \int \frac{2}{3}x e^{3x} \, dx$$

for $\int \frac{2}{3}x e^{3x} \, dx$, $u = \frac{2}{3}x$, $\frac{du}{dx} = \frac{2}{3}$; $\frac{dv}{dx} = e^{3x}$, $v = \frac{1}{3}e^{3x}$

$$\begin{aligned}\int \frac{2}{3}x e^{3x} \, dx &= \frac{2}{9}x e^{3x} - \int \frac{2}{9}e^{3x} \, dx \\ &= \frac{2}{9}x e^{3x} - \frac{2}{27}e^{3x} + c\end{aligned}$$

$$\begin{aligned}\therefore \int x^2 e^{3x} \, dx &= \frac{1}{3}x^2 e^{3x} - \left(\frac{2}{9}x e^{3x} - \frac{2}{27}e^{3x}\right) + c \\ &= \frac{1}{27}e^{3x}(9x^2 - 6x + 2) + c\end{aligned}$$

c $u = e^{-x}$, $\frac{du}{dx} = -e^{-x}$; $\frac{dv}{dx} = \cos 2x$, $v = \frac{1}{2} \sin 2x$

$$\begin{aligned}\int e^{-x} \cos 2x \, dx &= \frac{1}{2}e^{-x} \sin 2x - \int -\frac{1}{2}e^{-x} \sin 2x \, dx \\ &= \frac{1}{2}e^{-x} \sin 2x + \int \frac{1}{2}e^{-x} \sin 2x \, dx\end{aligned}$$

for $\int \frac{1}{2}e^{-x} \sin 2x \, dx$, $u = \frac{1}{2}e^{-x}$, $\frac{du}{dx} = -\frac{1}{2}e^{-x}$; $\frac{dv}{dx} = \sin 2x$, $v = -\frac{1}{4} \cos 2x$

$$\int \frac{1}{2}e^{-x} \sin 2x \, dx = -\frac{1}{4}e^{-x} \cos 2x - \int \frac{1}{4}e^{-x} \cos 2x \, dx$$

$$\therefore \int e^{-x} \cos 2x \, dx = \frac{1}{2}e^{-x} \sin 2x - \frac{1}{4}e^{-x} \cos 2x - \frac{1}{4} \int e^{-x} \cos 2x \, dx$$

$$\frac{5}{4} \int e^{-x} \cos 2x \, dx = \frac{1}{2}e^{-x} \sin 2x - \frac{1}{4}e^{-x} \cos 2x + c$$

$$\int e^{-x} \cos 2x \, dx = \frac{1}{5}e^{-x}(2 \sin 2x - \cos 2x) + c$$

8 a $\frac{1}{x}$

b $u = \ln x$, $\frac{du}{dx} = \frac{1}{x}$; $\frac{dv}{dx} = 1$, $v = x$

$$\begin{aligned}\int \ln x \, dx &= x \ln x - \int \frac{1}{x} \times x \, dx \\ &= x \ln x - \int dx \\ &= x \ln x - x + c \\ &= x(\ln x - 1) + c\end{aligned}$$

9 a $u = \ln 2x, \frac{du}{dx} = \frac{1}{x}; \frac{dv}{dx} = 1, v = x$

$$\begin{aligned}\int \ln 2x \, dx &= x \ln 2x - \int \frac{1}{x} \times x \, dx \\ &= x \ln 2x - \int dx \\ &= x \ln 2x - x + c \\ &= x(\ln 2x - 1) + c\end{aligned}$$

b $u = \ln x, \frac{du}{dx} = \frac{1}{x}; \frac{dv}{dx} = 3x, v = \frac{3}{2}x^2$

$$\begin{aligned}\int 3x \ln x \, dx &= \frac{3}{2}x^2 \ln x - \int \frac{1}{x} \times \frac{3}{2}x^2 \, dx \\ &= \frac{3}{2}x^2 \ln x - \int \frac{3}{2}x \, dx \\ &= \frac{3}{2}x^2 \ln x - \frac{3}{4}x^2 + c \\ &= \frac{3}{4}x^2(2 \ln x - 1) + c\end{aligned}$$

c $u = (\ln x)^2, \frac{du}{dx} = 2(\ln x) \times \frac{1}{x}; \frac{dv}{dx} = 1, v = x$

$$\int (\ln x)^2 \, dx = x(\ln x)^2 - \int 2 \ln x \, dx$$

for $\int 2 \ln x \, dx, u = \ln x, \frac{du}{dx} = \frac{1}{x}; \frac{dv}{dx} = 2, v = 2x$

$$\begin{aligned}\int 2 \ln x \, dx &= 2x \ln x - \int 2 \, dx \\ &= 2x \ln x - 2x + c\end{aligned}$$

$$\begin{aligned}\therefore \int (\ln x)^2 \, dx &= x(\ln x)^2 - (2x \ln x - 2x) + c \\ &= x[(\ln x)^2 - 2 \ln x + 2] + c\end{aligned}$$

10 a $u = x + 2, \frac{du}{dx} = 1; \frac{dv}{dx} = e^x, v = e^x$

$$\begin{aligned}\int_{-1}^0 (x+2)e^x \, dx &= [(x+2)e^x]_{-1}^0 - \int_{-1}^0 e^x \, dx \\ &= [(x+2)e^x - e^x]_{-1}^0 \\ &= (2-1) - (e^{-1} - e^{-1}) \\ &= 1\end{aligned}$$

b $u = \ln x, \frac{du}{dx} = \frac{1}{x}; \frac{dv}{dx} = x^2, v = \frac{1}{3}x^3$

$$\begin{aligned}\int_1^2 x^2 \ln x \, dx &= \left[\frac{1}{3}x^3 \ln x\right]_1^2 - \int_1^2 \frac{1}{3}x^2 \, dx \\ &= \left[\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3\right]_1^2 \\ &= \left(\frac{8}{3} \ln 2 - \frac{8}{9}\right) - \left(0 - \frac{1}{9}\right) \\ &= \frac{8}{3} \ln 2 - \frac{7}{9}\end{aligned}$$

c $u = 2x, \frac{du}{dx} = 2; \frac{dv}{dx} = e^{3x-1}, v = \frac{1}{3}e^{3x-1}$

$$\begin{aligned}\int_{\frac{1}{3}}^1 2xe^{3x-1} \, dx &= \left[\frac{2}{3}xe^{3x-1}\right]_{\frac{1}{3}}^1 - \int_{\frac{1}{3}}^1 \frac{2}{3}e^{3x-1} \, dx \\ &= \left[\frac{2}{3}xe^{3x-1} - \frac{2}{9}e^{3x-1}\right]_{\frac{1}{3}}^1 \\ &= \left(\frac{2}{3}e^2 - \frac{2}{9}e^2\right) - \left(\frac{2}{9} - \frac{2}{9}\right) \\ &= \frac{4}{9}e^2\end{aligned}$$

d $u = \ln(2x+3), \frac{du}{dx} = \frac{2}{2x+3}; \frac{dv}{dx} = 1, v = x$

$$\begin{aligned}\int_0^3 \ln(2x+3) \, dx &= [x \ln(2x+3)]_0^3 - \int_0^3 \frac{2x}{2x+3} \, dx \\ &= [x \ln(2x+3)]_0^3 - \int_0^3 \frac{(2x+3)-3}{2x+3} \, dx \\ &= [x \ln(2x+3)]_0^3 - \int_0^3 \left(1 - \frac{3}{2x+3}\right) \, dx \\ &= [x \ln(2x+3) - x + \frac{3}{2} \ln|2x+3|]_0^3 \\ &= (3 \ln 9 - 3 + \frac{3}{2} \ln 9) - (0 - 0 + \frac{3}{2} \ln 3) \\ &= \frac{15}{2} \ln 3 - 3\end{aligned}$$

$$\mathbf{e} \quad u = x^2, \quad \frac{du}{dx} = 2x; \quad \frac{dv}{dx} = \cos x, \quad v = \sin x$$

$$\int x^2 \cos x \, dx = x^2 \sin x - \int 2x \sin x \, dx$$

$$\text{for } \int 2x \sin x \, dx, \quad u = 2x, \quad \frac{du}{dx} = 2; \quad \frac{dv}{dx} = \sin x, \quad v = -\cos x$$

$$\begin{aligned} \int 2x \sin x \, dx &= -2x \cos x - \int -2 \cos x \, dx \\ &= -2x \cos x + \int 2 \cos x \, dx \\ &= -2x \cos x + 2 \sin x + c \end{aligned}$$

$$\begin{aligned} \therefore \int_0^{\frac{\pi}{2}} x^2 \cos x \, dx &= [x^2 \sin x + 2x \cos x - 2 \sin x]_0^{\frac{\pi}{2}} \\ &= \left(\frac{1}{4}\pi^2 + 0 - 2\right) - (0 + 0 - 0) \\ &= \frac{1}{4}\pi^2 - 2 \end{aligned}$$

$$\mathbf{f} \quad u = e^{3x}, \quad \frac{du}{dx} = 3e^{3x}; \quad \frac{dv}{dx} = \sin 2x, \quad v = -\frac{1}{2} \cos 2x$$

$$\begin{aligned} \int e^{3x} \sin 2x \, dx &= -\frac{1}{2}e^{3x} \cos 2x - \int -\frac{3}{2}e^{3x} \cos 2x \, dx \\ &= -\frac{1}{2}e^{3x} \cos 2x + \int \frac{3}{2}e^{3x} \cos 2x \, dx \end{aligned}$$

$$\text{for } \int \frac{3}{2}e^{3x} \cos 2x \, dx, \quad u = \frac{3}{2}e^{3x}, \quad \frac{du}{dx} = \frac{9}{2}e^{3x}; \quad \frac{dv}{dx} = \cos 2x, \quad v = \frac{1}{2} \sin 2x$$

$$\int \frac{3}{2}e^{3x} \cos 2x \, dx = \frac{3}{4}e^{3x} \sin 2x - \int \frac{9}{4}e^{3x} \sin 2x \, dx$$

$$\begin{aligned} \therefore \int e^{3x} \sin 2x \, dx &= -\frac{1}{2}e^{3x} \cos 2x + \frac{3}{4}e^{3x} \sin 2x - \int \frac{9}{4}e^{3x} \sin 2x \, dx \\ \frac{13}{4} \int e^{3x} \sin 2x \, dx &= -\frac{1}{2}e^{3x} \cos 2x + \frac{3}{4}e^{3x} \sin 2x + c \end{aligned}$$

$$\begin{aligned} \therefore \int_0^{\frac{\pi}{4}} e^{3x} \sin 2x \, dx &= \frac{4}{13} \left[-\frac{1}{2}e^{3x} \cos 2x + \frac{3}{4}e^{3x} \sin 2x \right]_0^{\frac{\pi}{4}} \\ &= \frac{4}{13} \left[\left(0 + \frac{3}{4}e^{\frac{3\pi}{4}}\right) - \left(-\frac{1}{2} + 0\right) \right] \\ &= \frac{1}{13} (3e^{\frac{3\pi}{4}} + 2) \end{aligned}$$