

1 a Express  $\frac{x+4}{(1+x)(2-x)}$  in partial fractions.

b Given that  $y = 2$  when  $x = 3$ , solve the differential equation

$$\frac{dy}{dx} = \frac{y(x+4)}{(1+x)(2-x)}.$$

2 Given that  $y = 0$  when  $x = 0$ , solve the differential equation

$$\frac{dy}{dx} = e^{x+y} \cos x.$$

3 Given that  $\frac{dy}{dx}$  is inversely proportional to  $x$  and that  $y = 4$  and  $\frac{dy}{dx} = \frac{5}{3}$  when  $x = 3$ , find an expression for  $y$  in terms of  $x$ .

4 A quantity has the value  $N$  at time  $t$  hours and is increasing at a rate proportional to  $N$ .

a Write down a differential equation relating  $N$  and  $t$ .

b By solving your differential equation, show that

$$N = Ae^{kt},$$

where  $A$  and  $k$  are constants and  $k$  is positive.

Given that when  $t = 0$ ,  $N = 40$  and that when  $t = 5$ ,  $N = 60$ ,

c find the values of  $A$  and  $k$ ,

d find the value of  $N$  when  $t = 12$ .

5 A cube is increasing in size and has volume  $V \text{ cm}^3$  and surface area  $A \text{ cm}^2$  at time  $t$  seconds.

a Show that

$$\frac{dV}{dA} = k\sqrt{A},$$

where  $k$  is a positive constant.

Given that the rate at which the volume of the cube is increasing is proportional to its surface area

and that when  $t = 10$ ,  $A = 100$  and  $\frac{dA}{dt} = 5$ ,

b show that

$$A = \frac{1}{16}(t + 30)^2.$$

6 At time  $t = 0$ , a piece of radioactive material has mass 24 g. Its mass after  $t$  days is  $m$  grams and is decreasing at a rate proportional to  $m$ .

a By forming and solving a suitable differential equation, show that

$$m = 24e^{-kt},$$

where  $k$  is a positive constant.

After 20 days, the mass of the material is found to be 22.6 g.

b Find the value of  $k$ .

c Find the rate at which the mass is decreasing after 20 days.

d Find how long it takes for the mass of the material to be halved.

7 A quantity has the value  $P$  at time  $t$  seconds and is decreasing at a rate proportional to  $\sqrt{P}$ .

a By forming and solving a suitable differential equation, show that

$$P = (a - bt)^2,$$

where  $a$  and  $b$  are constants.

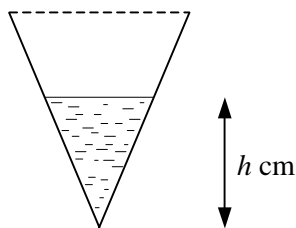
Given that when  $t = 0$ ,  $P = 400$ ,

b find the value of  $a$ .

Given also that when  $t = 30$ ,  $P = 100$ ,

c find the value of  $P$  when  $t = 50$ .

8



The diagram shows a container in the shape of a right-circular cone. A quantity of water is poured into the container but this then leaks out from a small hole at its vertex.

In a model of the situation it is assumed that the rate at which the volume of water in the container,  $V \text{ cm}^3$ , decreases is proportional to  $V$ . Given that the depth of the water is  $h \text{ cm}$  at time  $t$  minutes,

a show that

$$\frac{dh}{dt} = -kh,$$

where  $k$  is a positive constant.

Given also that  $h = 12$  when  $t = 0$  and that  $h = 10$  when  $t = 20$ ,

b show that

$$h = 12e^{-kt},$$

and find the value of  $k$ ,

c find the value of  $t$  when  $h = 6$ .

9 a Express  $\frac{1}{(1+x)(1-x)}$  in partial fractions.

In an industrial process, the mass of a chemical,  $m \text{ kg}$ , produced after  $t$  hours is modelled by the differential equation

$$\frac{dm}{dt} = ke^{-t}(1+m)(1-m),$$

where  $k$  is a positive constant.

Given that when  $t = 0$ ,  $m = 0$  and that the initial rate at which the chemical is produced is  $0.5 \text{ kg per hour}$ ,

b find the value of  $k$ ,

c show that, for  $0 \leq m < 1$ ,  $\ln \left( \frac{1+m}{1-m} \right) = 1 - e^{-t}$ .

d find the time taken to produce  $0.1 \text{ kg}$  of the chemical,

e show that however long the process is allowed to run, the maximum amount of the chemical that will be produced is about  $462 \text{ g}$ .