

$$1 \quad \mathbf{a} \quad \frac{1}{x^2-3x+2} \equiv \frac{A}{x-1} + \frac{B}{x-2}$$

$$1 \equiv A(x-2) + B(x-1)$$

$$x=1 \Rightarrow 1 = -A \Rightarrow A = -1$$

$$x=2 \quad \Rightarrow B = 1$$

$$\therefore \frac{1}{x^2-3x+2} \equiv \frac{1}{x-2} - \frac{1}{x-1}$$

$$\begin{aligned} \mathbf{b} \quad \int_3^4 \frac{1}{x^2-3x+2} dx &= \int_3^4 \left(\frac{1}{x-2} - \frac{1}{x-1} \right) dx \\ &= [\ln|x-2| - \ln|x-1|]_3^4 \\ &= (\ln 2 - \ln 3) - (0 - \ln 2) \\ &= 2 \ln 2 - \ln 3 = \ln \frac{2^2}{3} \\ &= \ln \frac{4}{3} \quad [a=4, b=3] \end{aligned}$$

$$3 \quad \mathbf{a} \quad \begin{array}{r} x+2 \\ x-1 \overline{) x^2+x-1} \\ \underline{x^2-x} \\ 2x-1 \\ \underline{2x-2} \\ 1 \end{array}$$

quotient: $x+2$, remainder: 1

$$\begin{aligned} \mathbf{b} \quad \int \frac{x^2+x-1}{x-1} dx &= \int \left(x+2 + \frac{1}{x-1} \right) dx \\ &= \frac{1}{2}x^2 + 2x + \ln|x-1| + c \end{aligned}$$

$$5 \quad \mathbf{a} \quad u = \ln x, \frac{du}{dx} = \frac{1}{x}; \quad \frac{dv}{dx} = x, v = \frac{1}{2}x^2$$

$$\begin{aligned} \int x \ln x dx &= \frac{1}{2}x^2 \ln x - \int \frac{1}{2}x dx \\ &= \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + c \\ &= \frac{1}{4}x^2(2 \ln x - 1) + c \end{aligned}$$

$$\mathbf{b} \quad \int \frac{1}{y} dy = \int x \ln x dx$$

$$\ln|y| = \frac{1}{4}x^2(2 \ln x - 1) + c$$

$$y > 0 \quad \therefore \ln y = \frac{1}{4}x^2(2 \ln x - 1) + c$$

$$y = 4 \quad \text{when } x = 2$$

$$\therefore \ln 4 = 2 \ln 2 - 1 + c, \quad c = 1$$

$$\therefore \ln y = \frac{1}{4}x^2(2 \ln x - 1) + 1$$

$$\text{when } x = 1, \ln y = \frac{1}{4}(0 - 1) + 1 = \frac{3}{4}$$

$$\therefore y = e^{\frac{3}{4}}$$

$$2 \quad = \int_0^{\frac{\pi}{6}} \frac{1}{2} [\cos 4x + \cos(-2x)] dx$$

$$= \int_0^{\frac{\pi}{6}} \left(\frac{1}{2} \cos 4x + \frac{1}{2} \cos 2x \right) dx$$

$$= \left[\frac{1}{8} \sin 4x + \frac{1}{4} \sin 2x \right]_0^{\frac{\pi}{6}}$$

$$= \left(\frac{1}{8} \times \frac{\sqrt{3}}{2} + \frac{1}{4} \times \frac{\sqrt{3}}{2} \right) - (0)$$

$$= \frac{3}{16} \sqrt{3}$$

$$6 \quad \mathbf{a} \quad = \int_0^{\frac{\pi}{3}} \sec x \tan x dx$$

$$= [\sec x]_0^{\frac{\pi}{3}}$$

$$= 2 - 1$$

$$= 1$$

$$\mathbf{b} \quad u = \cos \theta \quad \therefore \frac{du}{d\theta} = -\sin \theta$$

$$\theta = 0 \Rightarrow u = 1$$

$$\theta = \frac{\pi}{4} \Rightarrow u = \frac{1}{\sqrt{2}}$$

$$\int_0^{\frac{\pi}{4}} \frac{\sin \theta}{\cos^4 \theta} d\theta = \int_1^{\frac{1}{\sqrt{2}}} \frac{1}{u^4} \times (-1) du$$

$$= \int_{\frac{1}{\sqrt{2}}}^1 u^{-4} du$$

$$= \left[-\frac{1}{3} u^{-3} \right]_{\frac{1}{\sqrt{2}}}^1$$

$$= -\frac{1}{3} (1 - 2\sqrt{2})$$

$$= -\frac{1}{3} + \frac{2}{3}\sqrt{2} \quad [a = -\frac{1}{3}, b = \frac{2}{3}]$$

$$8 \quad \mathbf{a} \quad u = 6x, \frac{du}{dx} = 6; \frac{dv}{dx} = \cos 3x, v = \frac{1}{3} \sin 3x$$

$$\int 6x \cos 3x \, dx = 2x \sin 3x - \int 2 \sin 3x \, dx$$

$$= 2x \sin 3x + \frac{2}{3} \cos 3x + c$$

$$\mathbf{b} \quad x = 2 \sin u \quad \therefore \frac{dx}{du} = 2 \cos u$$

$$x = 0 \Rightarrow u = 0$$

$$x = \sqrt{3} \Rightarrow u = \frac{\pi}{3}$$

$$\int_0^{\sqrt{3}} \frac{1}{\sqrt{4-x^2}} \, dx = \int_0^{\frac{\pi}{3}} \frac{1}{2 \cos u} \times 2 \cos u \, du$$

$$= \int_0^{\frac{\pi}{3}} du$$

$$= [u]_0^{\frac{\pi}{3}}$$

$$= \frac{\pi}{3} - 0$$

$$= \frac{\pi}{3}$$

$$9 \quad \mathbf{a} \quad A = \pi r^2 \quad \therefore r = \sqrt{\frac{A}{\pi}}$$

$$P = 2\pi r = 2\pi \sqrt{\frac{A}{\pi}} = 2\sqrt{\pi A}$$

$$\frac{dA}{dt} = cP = c \times 2\sqrt{\pi A} = 2c\sqrt{\pi} \times \sqrt{A}$$

$$\therefore \frac{dA}{dt} = k\sqrt{A}$$

$$\mathbf{b} \quad \int A^{-\frac{1}{2}} \, dA = \int k \, dt$$

$$2A^{\frac{1}{2}} = kt + C$$

$$\sqrt{A} = \frac{1}{2}kt + \frac{1}{2}C = pt + q$$

$$\therefore A = (pt + q)^2$$

$$\mathbf{c} \quad t = 0, A = 25 \quad \therefore \sqrt{25} = 0 + q$$

$$q = 5$$

$$t = 20, A = 40 \quad \therefore \sqrt{40} = 20p + 5$$

$$p = \frac{2\sqrt{10}-5}{20}$$

$$A = 50 \Rightarrow 50 = \left(\frac{2\sqrt{10}-5}{20}t + 5\right)^2$$

$$t = (5\sqrt{2} - 5) \div \frac{2\sqrt{10}-5}{20}$$

$$= 31.3 \text{ minutes (3sf)}$$

$$10 \quad \mathbf{a} \quad \frac{5x+1}{(1-x)(1+2x)} \equiv \frac{A}{1-x} + \frac{B}{1+2x}$$

$$5x + 1 \equiv A(1 + 2x) + B(1 - x)$$

$$x = 1 \Rightarrow 6 = 3A \Rightarrow A = 2$$

$$x = -\frac{1}{2} \Rightarrow -\frac{3}{2} = \frac{3}{2}B \Rightarrow B = -1$$

$$f(x) \equiv \frac{2}{1-x} - \frac{1}{1+2x}$$

$$\mathbf{b} \quad = \int_0^{\frac{1}{2}} \left(\frac{2}{1-x} - \frac{1}{1+2x}\right) dx$$

$$= [-2 \ln |1-x| - \frac{1}{2} \ln |1+2x|]_0^{\frac{1}{2}}$$

$$= (-2 \ln \frac{1}{2} - \frac{1}{2} \ln 2) - (0 - 0)$$

$$= 2 \ln 2 - \frac{1}{2} \ln 2 = \frac{3}{2} \ln 2$$

$$\mathbf{c} \quad (1-x)^{-1} =$$

$$1 + (-1)(-x) + \frac{(-1)(-2)}{2} (-x)^2 + \frac{(-1)(-2)(-3)}{3 \times 2} (-x)^3 + \dots$$

$$= 1 + x + x^2 + x^3 + \dots$$

$$(1+2x)^{-1} =$$

$$1 + (-1)(2x) + \frac{(-1)(-2)}{2} (2x)^2 + \frac{(-1)(-2)(-3)}{3 \times 2} (2x)^3 + \dots$$

$$= 1 - 2x + 4x^2 - 8x^3 + \dots$$

$$f(x) =$$

$$2(1 + x + x^2 + x^3) - (1 - 2x + 4x^2 - 8x^3) + \dots$$

$$f(x) = 1 + 4x - 2x^2 + 10x^3 + \dots$$