

1 a Express  $\frac{1}{x^2-3x+2}$  in partial fractions. (3)

b Show that

$$\int_3^4 \frac{1}{x^2-3x+2} dx = \ln \frac{a}{b},$$

where  $a$  and  $b$  are integers to be found. (5)

2 Evaluate

$$\int_0^{\frac{\pi}{6}} \cos x \cos 3x dx. \quad (6)$$

3 a Find the quotient and remainder obtained in dividing  $(x^2 + x - 1)$  by  $(x - 1)$ . (3)

b Hence, show that

$$\int \frac{x^2+x-1}{x-1} dx = \frac{1}{2}x^2 + 2x + \ln|x-1| + c,$$

where  $c$  is an arbitrary constant. (2)

5 a Use integration by parts to find  $\int x \ln x dx$ . (4)

b Given that  $y = 4$  when  $x = 2$ , solve the differential equation

$$\frac{dy}{dx} = xy \ln x, \quad x > 0, \quad y > 0,$$

and hence, find the exact value of  $y$  when  $x = 1$ . (5)

6 a Evaluate  $\int_0^{\frac{\pi}{3}} \sin x \sec^2 x dx$ . (4)

b Using the substitution  $u = \cos \theta$ , or otherwise, show that

$$\int_0^{\frac{\pi}{4}} \frac{\sin \theta}{\cos^4 \theta} d\theta = a + b\sqrt{2},$$

where  $a$  and  $b$  are rational. (6)

- 8 a Using integration by parts, find

$$\int 6x \cos 3x \, dx. \quad (5)$$

- b Use the substitution  $x = 2 \sin u$  to show that

$$\int_0^{\sqrt{3}} \frac{1}{\sqrt{4-x^2}} \, dx = \frac{\pi}{3}. \quad (5)$$

- 9 In an experiment to investigate the formation of ice on a body of water, a thin circular disc of ice is placed on the surface of a tank of water and the surrounding air temperature is kept constant at  $-5^\circ\text{C}$ .

In a model of the situation, it is assumed that the disc of ice remains circular and that its area,  $A \text{ cm}^2$  after  $t$  minutes, increases at a rate proportional to its perimeter.

- a Show that

$$\frac{dA}{dt} = k\sqrt{A},$$

where  $k$  is a positive constant. (3)

- b Show that the general solution of this differential equation is

$$A = (pt + q)^2,$$

where  $p$  and  $q$  are constants. (4)

Given that when  $t = 0$ ,  $A = 25$  and that when  $t = 20$ ,  $A = 40$ ,

- c find how long it takes for the area to increase to  $50 \text{ cm}^2$ . (5)

10 
$$f(x) \equiv \frac{5x+1}{(1-x)(1+2x)}.$$

- a Express  $f(x)$  in partial fractions. (3)

- b Find  $\int_0^{\frac{1}{2}} f(x) \, dx$ , giving your answer in the form  $k \ln 2$ . (4)

- c Find the series expansion of  $f(x)$  in ascending powers of  $x$  up to and including the term in  $x^3$ , for  $|x| < \frac{1}{2}$ . (6)