

2 a	x	0	0.5	1	1.5	2	2.5	3	3.5	4
	$x \sec(\frac{1}{3}x)$	0	0.507	1.058	1.709	2.545	3.718	5.552	8.901	17.004

$$\text{i} \approx \frac{1}{2} \times 2 \times [0 + 17.004 + 2(2.545)] = 22.1 \text{ (3sf)}$$

$$\text{ii} \approx \frac{1}{2} \times 1 \times [0 + 17.004 + 2(1.058 + 2.545 + 5.552)] = 17.7 \text{ (3sf)}$$

$$\text{iii} \approx \frac{1}{2} \times 0.5 \times [0 + 17.004 + 2(0.507 + 1.058 + 1.709 + 2.545 + 3.718 + 5.552 + 8.901)] = 16.2 \text{ (3sf)}$$

- b** e.g. halving the interval width leads to the estimate reducing by 4.4, then 1.5
further halving might lead to reductions of approximately 0.5 and 0.2 so $I \approx 15.5$

$$\mathbf{3 \ a} \quad -\frac{d\theta}{dt} = k\theta$$

$$\int \frac{1}{\theta} d\theta = \int -k dt$$

$$\ln |\theta| = -kt + c$$

$$\theta = e^{-kt+c} = e^c \times e^{-kt}$$

$$\therefore \theta = Ae^{-kt}$$

$$t = 0, \theta = 25 - 10 = 15$$

$$\therefore A = 15$$

$$\therefore \theta = 15e^{-kt}$$

$$\mathbf{b} \quad t = 30, \theta = 20 - 10 = 10$$

$$\therefore 10 = 15e^{-30k}$$

$$k = -\frac{1}{30} \ln \frac{2}{3} = 0.0135 \text{ (3sf)}$$

$$\mathbf{c} \quad \theta = 15 - 10 = 5$$

$$\therefore 5 = 15e^{-0.01352t}$$

$$t = -\frac{1}{0.01352} \ln \frac{1}{3}$$

$$= 81.3 \text{ minutes (3sf)}$$

$$\mathbf{4 \ a} \quad \sin^4 x \equiv (\sin^2 x)^2$$

$$\equiv \left[\frac{1}{2}(1 - \cos 2x) \right]^2$$

$$\equiv \frac{1}{4}(1 - 2\cos 2x + \cos^2 2x)$$

$$\equiv \frac{1}{4} - \frac{1}{2}\cos 2x + \frac{1}{4} \left[\frac{1}{2}(1 + \cos 4x) \right]$$

$$\equiv \frac{3}{8} - \frac{1}{2}\cos 2x + \frac{1}{8}\cos 4x$$

$$\therefore p = \frac{3}{8}, q = -\frac{1}{2}, r = \frac{1}{8}$$

$$\mathbf{b} \quad = \int_0^{\frac{\pi}{2}} \left(\frac{3}{8} - \frac{1}{2}\cos 2x + \frac{1}{8}\cos 4x \right) dx$$

$$= \left[\frac{3}{8}x - \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x \right]_0^{\frac{\pi}{2}}$$

$$= \left(\frac{3}{16}\pi - 0 + 0 \right) - (0 - 0 + 0)$$

$$= \frac{3}{16}\pi$$

$$5 \quad \mathbf{a} \quad \int y^{-3} dy = \int x dx$$

$$-\frac{1}{2}y^{-2} = \frac{1}{2}x^2 + c$$

$$y^{-2} = k - x^2$$

$$y^2 = \frac{1}{k-x^2}$$

$$\mathbf{b} \quad y = \frac{1}{2} \text{ when } x = 1$$

$$\therefore \frac{1}{4} = \frac{1}{k-1}$$

$$k = 5$$

$$\therefore y^2 = \frac{1}{5-x^2}$$

$$6 \quad \mathbf{a} \quad x = e^u \therefore u = \ln x, \quad \frac{dx}{du} = e^u$$

$$\int \frac{2+\ln x}{x^2} dx = \int \frac{2+u}{(e^u)^2} \times e^u du$$

$$= \int (2+u)e^{-u} du$$

$$\mathbf{b} \quad x = e^u$$

$$x = 1 \Rightarrow u = 0$$

$$x = e \Rightarrow u = 1$$

$$\therefore \int_1^e \frac{2+\ln x}{x^2} dx = \int_0^1 (2+u)e^{-u} du$$

$$v = 2+u, \quad \frac{dv}{du} = 1; \quad \frac{dw}{du} = e^{-u}, \quad w = -e^{-u}$$

$$= [-(2+u)e^{-u}]_0^1 + \int_0^1 e^{-u} du$$

$$= [-(2+u)e^{-u} - e^{-u}]_0^1$$

$$= (-3e^{-1} - e^{-1}) - (-2 - 1)$$

$$= 3 - 4e^{-1}$$

$$8 \quad \mathbf{a} \quad \frac{6-2x^2}{(x+1)^2(x+3)} \equiv \frac{A}{(x+1)^2} + \frac{B}{x+1} + \frac{C}{x+3}$$

$$6-2x^2 \equiv A(x+3) + B(x+1)(x+3) + C(x+1)^2$$

$$x = -1 \Rightarrow 4 = 2A \Rightarrow A = 2$$

$$x = -3 \Rightarrow -12 = 4C \Rightarrow C = -3$$

$$\text{coeffs } x^2 \Rightarrow -2 = B + C \Rightarrow B = 1$$

$$\mathbf{b} \quad x = 0 \Rightarrow y = 2$$

$$f(x) = 2(x+1)^{-2} + (x+1)^{-1} - 3(x+3)^{-1}$$

$$f'(x) = -4(x+1)^{-3} - (x+1)^{-2} + 3(x+3)^{-2}$$

$$\therefore f'(0) = -4 - 1 + \frac{1}{3} = -\frac{14}{3}$$

$$\therefore y - 2 = -\frac{14}{3}(x - 0)$$

$$3y - 6 = -14x$$

$$14x + 3y = 6$$

$$\mathbf{c} = \int_0^1 \left(\frac{2}{(x+1)^2} + \frac{1}{x+1} - \frac{3}{x+3} \right) dx$$

$$= [-2(x+1)^{-1} + \ln|x+1| - 3\ln|x+3|]_0^1$$

$$= (-1 + \ln 2 - 3\ln 4) - (-2 + 0 - 3\ln 3)$$

$$= 1 - 5\ln 2 + 3\ln 3$$