

- 1  $22 \equiv A(x+4) + B(2x-3)$   
 $x = \frac{3}{2} \Rightarrow 22 = \frac{11}{2}A \Rightarrow A = 4$   
 $x = -4 \Rightarrow 22 = -11B \Rightarrow B = -2$
- 2  $x+5 \equiv A(x-3)^2 + B(x+1)(x-3) + C(x+1)$   
 $x = -1 \Rightarrow 4 = 16A \Rightarrow A = \frac{1}{4}$   
 $x = 3 \Rightarrow 8 = 4C \Rightarrow C = 2$   
 coeffs of  $x^2 \Rightarrow 0 = A + B \Rightarrow B = -\frac{1}{4}$
- 3  $4x^2 - 16x - 7 \equiv A(2x-1)(x-4) + B(x-4) + C(2x-1)$   
 $x = 4 \Rightarrow -7 = 7C \Rightarrow C = -1$   
 $x = \frac{1}{2} \Rightarrow -14 = -\frac{7}{2}B \Rightarrow B = 4$   
 coeffs of  $x^2 \Rightarrow 4 = 2A \Rightarrow A = 2$

- 4 a  $f(1) = 3 + 11 + 8 - 4 = 18$   
 $f(-1) = -3 + 11 - 8 - 4 = -4$   
 $f(2) = 24 + 44 + 16 - 4 = 80$   
 $f(-2) = -24 + 44 - 16 - 4 = 0$   
 $\therefore (x+2)$  is a factor

$$\begin{array}{r} 3x^2 + 5x - 2 \\ x+2 \overline{) 3x^3 + 11x^2 + 8x - 4} \\ \underline{3x^3 + 6x^2} \phantom{- 4} \\ 5x^2 + 8x \phantom{- 4} \\ \underline{5x^2 + 10x} \phantom{- 4} \\ -2x - 4 \\ \underline{-2x - 4} \\ 0 \end{array}$$

$$\begin{aligned} \therefore f(x) &= (x+2)(3x^2 + 5x - 2) \\ &= (3x-1)(x+2)^2 \end{aligned}$$

- b  $\frac{x+16}{f(x)} \equiv \frac{A}{3x-1} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$   
 $x+16 \equiv A(x+2)^2 + B(3x-1)(x+2) + C(3x-1)$   
 $x = \frac{1}{3} \Rightarrow \frac{49}{3} = \frac{49}{9}A \Rightarrow A = 3$   
 $x = -2 \Rightarrow 14 = -7C \Rightarrow C = -2$   
 coeffs of  $x^2 \Rightarrow 0 = A + 3B \Rightarrow B = -1$   
 $\therefore \frac{x+16}{f(x)} \equiv \frac{3}{3x-1} - \frac{1}{x+2} - \frac{2}{(x+2)^2}$

- 5  $\frac{1}{x(2x-1)^2} \equiv \frac{A}{x} + \frac{B}{2x-1} + \frac{C}{(2x-1)^2}$   
 $1 \equiv A(2x-1)^2 + Bx(2x-1) + Cx$   
 $x = 0 \Rightarrow 1 = -C \Rightarrow C = -1$   
 $x = \frac{1}{2} \Rightarrow 1 = \frac{1}{2}C \Rightarrow C = 2$   
 coeffs of  $x^2 \Rightarrow 0 = 4A + 2B \Rightarrow B = -2$   
 $\therefore f(x) = \frac{1}{x} - \frac{2}{2x-1} + \frac{2}{(2x-1)^2}$

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$$\begin{array}{r}
 x^2 + 7x + 10 \overline{) \begin{array}{r} x^3 + 5x^2 - 2x - 19 \\ x^3 + 7x^2 + 10x \\ \hline - 2x^2 - 12x - 19 \\ - 2x^2 - 14x - 20 \\ \hline 2x + 1 \end{array} }
 \end{array}$$

$$\therefore f(x) \equiv x - 2 + \frac{2x+1}{x^2+7x+10}$$

$$\frac{2x+1}{(x+2)(x+5)} \equiv \frac{B}{x+2} + \frac{C}{x+5}$$

$$2x+1 \equiv B(x+5) + C(x+2)$$

$$x = -2 \Rightarrow -3 = 3B \Rightarrow B = -1$$

$$x = -5 \Rightarrow -9 = -3C \Rightarrow C = 3$$

$$\therefore f(x) \equiv x - 2 - \frac{1}{x+2} + \frac{3}{x+5} \quad [A = -2, B = -1, C = 3]$$

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$$\mathbf{a} \quad \frac{4}{(x+1)(x-1)} \equiv \frac{A}{x+1} + \frac{B}{x-1}$$

$$4 \equiv A(x-1) + B(x+1)$$

$$x = -1 \Rightarrow 4 = -2A \Rightarrow A = -2$$

$$x = 1 \Rightarrow 4 = 2B \Rightarrow B = 2$$

$$\therefore f(x) \equiv \frac{2}{x-1} - \frac{2}{x+1}$$

$$\mathbf{b} \quad \frac{2+5x-x^2}{(x-4)(x-2)(x-1)} \equiv \frac{A}{x-4} + \frac{B}{x-2} + \frac{C}{x-1}$$

$$2+5x-x^2 \equiv A(x-2)(x-1) + B(x-4)(x-1) + C(x-4)(x-2)$$

$$x = 4 \Rightarrow 6 = 6A \Rightarrow A = 1$$

$$x = 2 \Rightarrow 8 = -2B \Rightarrow B = -4$$

$$x = 1 \Rightarrow 6 = 3C \Rightarrow C = 2$$

$$\therefore g(x) \equiv \frac{1}{x-4} - \frac{4}{x-2} + \frac{2}{x-1}$$

$$\mathbf{c} \quad \frac{2}{x-1} - \frac{2}{x+1} = \frac{1}{x-4} - \frac{4}{x-2} + \frac{2}{x-1}$$

$$\frac{4}{x-2} - \frac{1}{x-4} - \frac{2}{x+1} = 0$$

$$\frac{4(x-4)(x+1) - (x-2)(x+1) - 2(x-2)(x-4)}{(x-2)(x-4)(x+1)} = 0$$

$$4(x^2 - 3x - 4) - (x^2 - x - 2) - 2(x^2 - 6x + 8) = 0$$

$$x^2 + x - 30 = 0$$

$$(x+6)(x-5) = 0$$

$$x = -6, 5$$