1 **a** $6\mathbf{i} + \mathbf{j}$ **b** $-4\mathbf{i} + 2\mathbf{j}$ **c** $-6\mathbf{i}$ **d** $10\mathbf{i} - 2\mathbf{j}$

2 **a** $= 4(\mathbf{i} - 3\mathbf{j})$
$= 4\mathbf{i} - 12\mathbf{j}$

b $= (4\mathbf{i} + 2\mathbf{j}) - (\mathbf{i} - 3\mathbf{j})$
$= 3\mathbf{i} + 5\mathbf{j}$

c $= 2(\mathbf{i} - 3\mathbf{j}) + 3(4\mathbf{i} + 2\mathbf{j})$
$= 14\mathbf{i}$

d $= 4(\mathbf{i} - 3\mathbf{j}) - 2(4\mathbf{i} + 2\mathbf{j})$
$= -4\mathbf{i} - 16\mathbf{j}$

3 **a** $= \sqrt{9+16} = 5$

b $= 2\sqrt{1+4} = 2\sqrt{5}$

c $\mathbf{p} + 2\mathbf{q} = \begin{pmatrix} 3 \\ -4 \end{pmatrix} + 2\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$
$|\,\mathbf{p} + 2\mathbf{q}\,| = 5$

d $3\mathbf{q} - 2\mathbf{p} = 3\begin{pmatrix} 1 \\ 2 \end{pmatrix} - 2\begin{pmatrix} 3 \\ -4 \end{pmatrix} = \begin{pmatrix} -3 \\ 14 \end{pmatrix}$
$|\,3\mathbf{q} - 2\mathbf{p}\,| = \sqrt{9+196} = \sqrt{205} = 14.3$ (3sf)

4 **a** $= \tan^{-1} \frac{1}{2} = 26.6°$

b $= \tan^{-1} 3 = 71.6°$

c $5\mathbf{p} + \mathbf{q} = 5(2\mathbf{i} + \mathbf{j}) + (\mathbf{i} - 3\mathbf{j}) = 11\mathbf{i} + 2\mathbf{j}$
angle $= \tan^{-1} \frac{2}{11} = 10.3°$

d $\mathbf{p} - 3\mathbf{q} = (2\mathbf{i} + \mathbf{j}) - 3(\mathbf{i} - 3\mathbf{j}) = -\mathbf{i} + 10\mathbf{j}$
angle $= 180° - \tan^{-1} 10 = 95.7°$

5 **a** $\left|\begin{pmatrix} 4 \\ 3 \end{pmatrix}\right| = \sqrt{16+9} = 5$
$\therefore\ \frac{1}{5}\begin{pmatrix} 4 \\ 3 \end{pmatrix}$

b $\left|\begin{pmatrix} 7 \\ -24 \end{pmatrix}\right| = \sqrt{49+576} = 25$
$\therefore\ \frac{1}{25}\begin{pmatrix} 7 \\ -24 \end{pmatrix}$

c $\left|\begin{pmatrix} -1 \\ 1 \end{pmatrix}\right| = \sqrt{1+1} = \sqrt{2}$
$\therefore\ \frac{1}{\sqrt{2}}\begin{pmatrix} -1 \\ 1 \end{pmatrix} = \frac{1}{2}\sqrt{2}\begin{pmatrix} -1 \\ 1 \end{pmatrix}$

d $\left|\begin{pmatrix} 2 \\ 4 \end{pmatrix}\right| = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$
$\therefore\ \frac{1}{2\sqrt{5}}\begin{pmatrix} 2 \\ 4 \end{pmatrix} = \frac{1}{5}\sqrt{5}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

6 **a** $|\,5\mathbf{i} + 12\mathbf{j}\,| = \sqrt{25+144} = 13$
$\therefore\ \frac{26}{13}(5\mathbf{i} + 12\mathbf{j}) = 10\mathbf{i} + 24\mathbf{j}$

b $|\,-6\mathbf{i} - 8\mathbf{j}\,| = \sqrt{36+64} = 10$
$\therefore\ \frac{15}{10}(-6\mathbf{i} - 8\mathbf{j}) = -9\mathbf{i} - 12\mathbf{j}$

c $|\,2\mathbf{i} - 4\mathbf{j}\,| = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$
$\therefore\ \frac{5}{2\sqrt{5}}(2\mathbf{i} - 4\mathbf{j}) = \sqrt{5}(\mathbf{i} - 2\mathbf{j})$

7 **a** $(2\mathbf{i} + \lambda\mathbf{j}) + (\mu\mathbf{i} - 5\mathbf{j}) = 3\mathbf{i} - \mathbf{j}$
$2 + \mu = 3$ and $\lambda - 5 = -1$
$\therefore\ \lambda = 4,\ \mu = 1$

b $2(2\mathbf{i} + \lambda\mathbf{j}) - (\mu\mathbf{i} - 5\mathbf{j}) = -3\mathbf{i} + 8\mathbf{j}$
$4 - \mu = -3$ and $2\lambda + 5 = 8$
$\therefore\ \lambda = \frac{3}{2},\ \mu = 7$

8 **a** $6\mathbf{i} + c\mathbf{j} = 3(2\mathbf{i} + \mathbf{j})$
$\therefore\ c = 3$

b $6\mathbf{i} + c\mathbf{j} = -\frac{2}{3}(-9\mathbf{i} - 6\mathbf{j})$
$\therefore\ c = 4$

c $36 + c^2 = 10^2 = 100$
$\therefore\ c^2 = 64$
$c > 0\ \therefore\ c = 8$

d $36 + c^2 = (3\sqrt{5})^2 = 45$
$\therefore\ c^2 = 9$
$c > 0\ \therefore\ c = 3$

9 **a** $a(\mathbf{i} + 3\mathbf{j}) + b(4\mathbf{i} - 2\mathbf{j}) = -5\mathbf{i} + 13\mathbf{j}$

$\therefore \quad a + 4b = -5 \quad (1)$

and $\quad 3a - 2b = 13 \quad (2)$

$(1) + 2\times(2) \quad \Rightarrow \quad 7a = 21$

$\therefore \; a = 3, \; b = -2$

b $c(\mathbf{i} + 3\mathbf{j}) + (4\mathbf{i} - 2\mathbf{j}) = k\mathbf{j}$

$\therefore \; c + 4 = 0$

$\therefore \; c = -4$

c $(\mathbf{i} + 3\mathbf{j}) + d(4\mathbf{i} - 2\mathbf{j}) = k(3\mathbf{i} - \mathbf{j})$

$\therefore \quad 1 + 4d = 3k$

and $\quad 3 - 2d = -k$

$(1) + 2\times(2) \quad \Rightarrow \quad 7 = k$

$\therefore \quad d = 5$

10 **a** $\overrightarrow{AB} = \begin{pmatrix} -5 \\ 2 \end{pmatrix} - \begin{pmatrix} 3 \\ 6 \end{pmatrix} = \begin{pmatrix} -8 \\ -4 \end{pmatrix}$

b $|\overrightarrow{AB}| = \sqrt{64+16} = \sqrt{80} = 4\sqrt{5}$

c $= \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB}$

$= \begin{pmatrix} 3 \\ 6 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} -8 \\ -4 \end{pmatrix} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$

d $\overrightarrow{OC} = \overrightarrow{AB}$

$\therefore$ pos. vector $= \begin{pmatrix} -8 \\ -4 \end{pmatrix}$