

- 1 Find the gradient at the point with  $x$ -coordinate 3 on each of the following curves.
- a**  $y = x^3$                       **b**  $y = 4x - x^2$                       **c**  $y = 2x^2 - 8x + 3$                       **d**  $y = \frac{3}{x} + 2$
- 2 Find the gradient of each curve at the given point.
- a**  $y = 3x^2 + x - 5$                       (1, -1)                      **b**  $y = x^4 + 2x^3$                       (-2, 0)  
**c**  $y = x(2x - 3)$                       (2, 2)                      **d**  $y = x^2 - 2x^{-1}$                       (2, 3)  
**e**  $y = x^2 + 6x + 8$                       (-3, -1)                      **f**  $y = 4x + x^{-2}$                       ( $\frac{1}{2}$ , 6)
- 3 Evaluate  $f'(4)$  when
- a**  $f(x) = (x + 1)^2$                       **b**  $f(x) = x^{\frac{1}{2}}$                       **c**  $f(x) = x - 4x^{-2}$                       **d**  $f(x) = 5 - 6x^{\frac{3}{2}}$
- 4 The curve with equation  $y = x^3 - 4x^2 + 3x$  crosses the  $x$ -axis at the points  $A$ ,  $B$  and  $C$ .
- a** Find the coordinates of the points  $A$ ,  $B$  and  $C$ .  
**b** Find the gradient of the curve at each of the points  $A$ ,  $B$  and  $C$ .
- 5 For the curve with equation  $y = 2x^2 - 5x + 1$ ,
- a** find  $\frac{dy}{dx}$ ,  
**b** find the value of  $x$  for which  $\frac{dy}{dx} = 7$ .
- 6 Find the coordinates of the points on the curve with the equation  $y = x^3 - 8x$  at which the gradient of the curve is 4.
- 7 A curve has the equation  $y = x^3 + x^2 - 4x + 1$ .
- a** Find the gradient of the curve at the point  $P(-1, 5)$ .  
Given that the gradient at the point  $Q$  on the curve is the same as the gradient at the point  $P$ ,  
**b** find, as exact fractions, the coordinates of the point  $Q$ .
- 8 Find an equation of the tangent to each curve at the given point.
- a**  $y = x^2$                       (2, 4)                      **b**  $y = x^2 + 3x + 4$                       (-1, 2)  
**c**  $y = 2x^2 - 6x + 8$                       (1, 4)                      **d**  $y = x^3 - 4x^2 + 2$                       (3, -7)
- 9 Find an equation of the tangent to each curve at the given point. Give your answers in the form  $ax + by + c = 0$ , where  $a$ ,  $b$  and  $c$  are integers.
- a**  $y = 3 - x^2$                       (-3, -6)                      **b**  $y = \frac{2}{x}$                       (2, 1)  
**c**  $y = 2x^2 + 5x - 1$                       ( $\frac{1}{2}$ , 2)                      **d**  $y = x - 3\sqrt{x}$                       (4, -2)
- 10 Find an equation of the normal to each curve at the given point. Give your answers in the form  $ax + by + c = 0$ , where  $a$ ,  $b$  and  $c$  are integers.
- a**  $y = x^2 - 4$                       (1, -3)                      **b**  $y = 3x^2 + 7x + 7$                       (-2, 5)  
**c**  $y = x^3 - 8x + 4$                       (2, -4)                      **d**  $y = x - \frac{6}{x}$                       (3, 1)

- 11** Find, in the form  $y = mx + c$ , an equation of
- the tangent to the curve  $y = 3x^2 - 5x + 2$  at the point on the curve with  $x$ -coordinate 2,
  - the normal to the curve  $y = x^3 + 5x^2 - 12$  at the point on the curve with  $x$ -coordinate  $-3$ .
- 12** A curve has the equation  $y = x^3 + 3x^2 - 16x + 2$ .
- Find an equation of the tangent to the curve at the point  $P(2, -10)$ .  
The tangent to the curve at the point  $Q$  is parallel to the tangent at the point  $P$ .
  - Find the coordinates of the point  $Q$ .
- 13** A curve has the equation  $y = x^2 - 3x + 4$ .
- Find an equation of the normal to the curve at the point  $A(2, 2)$ .  
The normal to the curve at  $A$  intersects the curve again at the point  $B$ .
  - Find the coordinates of the point  $B$ .
- 14**  $f(x) \equiv x^3 + 4x^2 - 18$ .
- Find  $f'(x)$ .
  - Show that the tangent to the curve  $y = f(x)$  at the point on the curve with  $x$ -coordinate  $-3$  passes through the origin.
- 15** The curve  $C$  has the equation  $y = 6 + x - x^2$ .
- Find the coordinates of the point  $P$ , where  $C$  crosses the positive  $x$ -axis, and the point  $Q$ , where  $C$  crosses the  $y$ -axis.
  - Find an equation of the tangent to  $C$  at  $P$ .
  - Find the coordinates of the point where the tangent to  $C$  at  $P$  meets the tangent to  $C$  at  $Q$ .
- 16** The straight line  $l$  is a tangent to the curve  $y = x^2 - 5x + 3$  at the point  $A$  on the curve.  
Given that  $l$  is parallel to the line  $3x + y = 0$ ,
- find the coordinates of the point  $A$ ,
  - find the equation of the line  $l$  in the form  $y = mx + c$ .
- 17** The line with equation  $y = 2x + k$  is a normal to the curve with equation  $y = \frac{16}{x^2}$ .  
Find the value of the constant  $k$ .