Name:

Date:

# Normal Distribution

## AS-Level

Edexcel

Mathematics

| Mark | Score (%) |
| --- | --- |
| ___ / 86 | |

### Materials

For this paper you must have:

- Ruler
- Pencil, Rubber, Protractor and Compass
- Scientific calculator, which you are expected to use when appropriate

### Instructions

- Answer all questions
- Answer questions in the space provided
- All working must be shown
- Do all rough work in this book. Cross out any rough work you don't want to be marked

### Information

- The marks for the questions are shown in brackets

Leave blank

**1** The random variable X has a B(60, 0.02) distribution. Use an appropriate approximation to find $P(X \leq 2)$ **(3)**

**(Total for question 1 is 3 marks)**

**2** A continuous random variable has a normal distribution with a mean 25.0 and standard deviation σ. The probability that any one observation of the random variable is greater than 20.0 is 0.75. Find the value of σ. **(4)**

**(Total for question 2 is 4 marks)**

**3** X is a continuous random variable.

(a) State two conditions needed for X to be well modelled by normal distribution. **(2)**

(b) It is given that $X \sim N(50.0, 8^2)$. The mean of 20 random observation of X is denoted X. Find $P(X > 47.0)$**.** **(4)**

**(Total for question 3 is 6 marks)**

**4** The random variable $G$ has mean 20.0 and standard deviation $\sigma$. It is given that $P(G > 15.0) = 0.6$. Assume that $G$ is normally distributed.

(a) Find the value of $\sigma$ **(4)**

(b) Given that $P(G > g) = 0.4$, find the value of $P(G > 2g)$. **(3)**

(c) It is known that no values of G are ever negative. State with a reason what this tells you about the assumption that G is normally distributed. **(4)**

**(Total for question 4 is 11 marks)**

**5** The random variable Y is normally distributed with mean $n$ and variance $y^2$. It is found that $P(Y > 150.0) = 0.0228$ and $P(Y > 143.0) = 0.9332$. Find the values of $n$ and $v$. **(6)**

**(Total for question 5 is 6 marks)**

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**6** The drug EPO (erythropoetin) is taken by some athletes to improve their performance. This drug is in fact banned and blood samples taken from athletes are tested to measure their 'hematocrit level'. If the level is over 50 it is considered that the athlete is likely to have taken EPO and the result is described as 'positive'. The measured hematocrit level of each athlete varies over time, even if EPO has not been taken.

For each athlete in a large population of innocent athletes, the variation in measured hematocrit level is described by the Normal distribution with mean 42.0 and standard deviation 3.0.

(a) Show that the probability that such an athlete tests positive for EPO in a random chosen test is 0.0038. **(3)**

(b) Find the probability that such an athlete tests positive on at least 1 of the 7 occasions during the year when hematocrit level is measured. (These occasions are spread at random through the year and all test results are assumed to be independent.) **(3)**

(c) It is standard policy to apply a penalty after testing positive. Comment briefly on this policy in the light of your answer to part (b). **(2)**

Suppose that 1000 tests are carried out on innocent athletes whose variation in measured hematocrit level is as described in first part of the question. It may be assumed that the probability of a positive result in each test is 0.0038, independently of all other test results.

(d) State the exact distribution of the number of positive tests **(2)**

(e) Use a suitable approximating distribution to find the probability that at least 10 tests are positive. **(4)**

(f) Because of genetic factors, a particular innocent athlete has an abnormally high natural hematocrit level. This athlete's measured level is Normally distributed with mean 48.0 and standard deviation 2.0. The usual limit of 50 for a positive test is to be altered for this athlete to a higher value h. Find the value of h for which this athlete would test positive on average just once in 200 occasions. **(4)**

**(Total for question 6 is 18 marks)**

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**7** It is known that, on average, 2 people in 5 in a certain country are overweight. A random sample of 400 people is chosen. Using a suitable approximation, find the probability that fewer than 165 people in the sample are overweight. **(5)**

**(Total for question 7 is 5 marks)**

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**8** A survey of adults in a certain large town found that 76% of people wore a watch on their left wrist, 15% wore a watch on their right wrist and 9% did not wear a watch.

(a) A random sample of 14 adults was taken. Find the probability that more than 2 adults did not wear a watch. **(3)**

(b) A random sample of 200 adults was taken. Using a suitable approximation, find the probability that more than 155 wore a watch on their left wrist. **(5)**

**(Total for question 8 is 8 marks)**

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**9** On any occasion when a particular gymnast perform a certain routine, the probability that she will perform it correctly is 0.65, independently of all other occasions

(a) Find the probability that she will perform the routine correctly on exactly 5 occasions out of 7. **(2)**

(b) On one day she performs the routine 50 times. Use a suitable approximation to estimate the probability that she will perform the routine correctly on fewer than 29 occasions. **(5)**

(c) One another day she performs the routine $n$ times. Find the smallest value of $n$ for which the expected number of correct performances is at least 8. **(2)**

**(Total for question 9 is 9 marks)**

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**10** In a certain country the time take for a common infection clear up is normally distributed with mean μ days and standard deviation 2.6 days. 25% of these infections clear up on less than 7 days.

(a) Find the value of μ. **(4)**

In another country the standard deviation of the time taken for the infection to clear up is the same in part (a) but the mean is 6.5 days. The time taken is normally distributed.

(b) Find the probability that, in a randomly chosen case from the country, the infection takes longer than 6.2 days to clear up. **(3)**

**(Total for question 10 is 7 marks)**

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**11** *Extralite* are testing a new long-life bulb. The lifetimes, in hours, are assumed to be normally distributed with mean μ and standard deviation σ. After extensive tests, they find that 19% of bulbs have a lifetime exceeding 5000 hours, while 5% have a lifetime under 4000hours.

(a) Illustrate this information on a sketch **(2)**

(b) Show that σ = 396 and find the value of μ. **(2)**

In the reminder of this question take μ to be 4650 and σ to be 400.

(c) Find the probability that a bulb chosen at random has lifetime between 4250 and 4750 hours. **(3)**

(d) *Extralite* wish to quote a lifetime which will be exceeded by 99% of bulbs. What time, correct to the nearest 100 hours, should they quite? **(3)**

A new school classroom has six light-fittings, each fitted with an *Extralite* long-life bilb.

(e) Find the probability that no more than one bulb needs to be replaced within the first 4250 hours of use. **(3)**

**(Total for question 11 is 9 marks)**