

Name:

Date:

Iteration

GCSE

Edexcel
Mathematics
Grade (9-1)

Mark

Score (%)

— 68

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Materials

For this paper you must have:

- Ruler
- Pencil, Rubber, Protractor and Compass
- Scientific calculator, which you are expected to use when appropriate

Instructions

- Answer all questions
- Answer questions in the space provided
- All working must be shown
- Do all rough work in this book. Cross out any rough work you don't want to be marked

Information

- The marks for the questions are shown in brackets

1 Using $x_{n+1} = x_n + 3$

With $x_0 = 2$

Find the values of x_1 , x_2 and x_3 .

$$x_1 = \dots\dots\dots$$

$$x_2 = \dots\dots\dots$$

$$x_3 = \dots\dots\dots$$

(Total for question 1 is 3 marks)

2 Using $x_{n+1} = 3 + \frac{9}{x_n^2}$

With $x_0 = 3$

Find the values of x_1 , x_2 and x_3 .

$$x_1 = \dots\dots\dots$$

$$x_2 = \dots\dots\dots$$

$$x_3 = \dots\dots\dots$$

(Total for question 2 is 3 marks)

3 Using $x_{n+1} = \frac{5}{x_n^2 + 3}$

With $x_0 = 1$

Find the values of x_1, x_2 and x_3 .

$x_1 = \dots\dots\dots$

$x_2 = \dots\dots\dots$

$x_3 = \dots\dots\dots$

(Total for question 3 is 3 marks)

4 Starting with $x_0 = 3$, use the iteration formula $x_{n+1} = \frac{7}{x_n^2} + 2$ three times to find an estimate for the solution to $x^3 - 2x^2 = 7$

$\dots\dots\dots$

(Total for question 2 is 3 marks)

- 5 The number of rabbits in a field t days from now is R_t where

$$R_0 = 220$$

$$R_{t+1} = 1.15(R_t - 20)$$

Work out the number of rabbits in the field 3 days from now.

.....
(Total for question 5 is 4 marks)

- 6 The number of people living in a town t years from now is P_t where

$$P_0 = 50000$$

$$P_{t+1} = 1.05(P_t - 800)$$

Work out the number of people in the town 3 years from now.

.....
(Total for question 6 is 4 marks)

7 (a) Complete the table for $y = x^3 - 5x - 4$

x	0	1	2	3	4
y		-8			40

(2)

(b) Between which two consecutive integers is there a solution to the equation $x^3 - 5x - 4 = 0$? Give a reason for your answer.

.....
(2)

(c) Show that the equation $x^3 - 5x - 4 = 0$ can be arranged to give $x = \sqrt[3]{5x + 4}$

.....
(1)

(d) Starting with $x_0 = 2.5$ use the iteration formula $x_{n+1} = \sqrt[3]{5x_n + 4}$ to find an estimate for the solution of $x^3 - 5x - 4 = 0$ to 2 decimal places.

.....
(3)

(Total for question 7 is 8 marks)

8 (a) Show that the equation $x^3 + 2x = 1$ has a solution between $x = 0$ and $x = 1$

(2)

(b) Show that the equation $x^3 + 2x = 1$ can be rearranged to give $x = \frac{1}{2} - \frac{x^3}{2}$

(1)

(c) Starting with $x_0 = 0$, use the iteration formula $x_{n+1} = \frac{1}{2} - \frac{x_n^3}{2}$ twice to find an estimate for the solution of $x^3 + 2x = 1$

(3)

(Total for question 8 is 6 marks)

9 (a) Show that the equation $3x - x^3 = -11$ has a solution between $x = 2$ and $x = 3$

(2)

(b) Show that the equation $3x - x^3 = -11$ can be rearranged to give $x = \sqrt[3]{3x + 11}$

(2)

(c) Starting with $x_0 = 3$, use the iteration formula $x_{n+1} = \sqrt[3]{3x_n + 11}$ three times to find an estimate for the solution of $3x - x^3 = -11$

.....

(3)

(Total for question 9 is 7 marks)

- 10 (a) Show that the equation $x^2 - x - 7 = 0$ can be rearranged to give $x = \sqrt{x+7}$

(2)

This information can be used to obtain the iterative formula $x_{n+1} = \sqrt{x_n + 7}$

- (b) Starting with $x = 4$ calculate the values of x_1, x_2 and x_3 , giving all the figures on your calculator display.

.....
(3)

- (c) Find one solution of $x^2 - x - 7 = 0$ correct to 3 decimal places.

.....
(1)

(Total for question 10 is 6 marks)

11 (a) Show that $x = 1 + \frac{11}{x-3}$ is a rearrangement of the equation $x^2 - 4x - 8 = 0$

(2)

(b) Use the iterative formula $x_{n+1} = 1 + \frac{11}{x_n - 3}$ together with a starting value of $x_0 = -2$ to obtain a solution of the equation $x^2 - 4x - 8 = 0$ correct to 1 decimal place.

.....
(3)

(Total for question 11 is 5 marks)

12 (a) Show that the equation $x^3 - 10x = 30$ has a solution between $x = 4$ and $x = 5$

(2)

(b) Show that the equation $x^3 - 10x = 30$ can be arranged to give $x = \sqrt[3]{30 + 10x}$

(1)

(c) Starting with $x_0 = 4.5$ use the iteration formula $x_{n+1} = \sqrt[3]{30 + 10x_n}$ to find an estimate for the solution of $x^3 - 10x = 30$ to 2 decimal places.

.....
(3)

(Total for question 12 is 6 marks)

13 Using $x_{n+1} = \frac{5}{x_n^2} + 2$

With $x_0 = 2.5$

(a) Find the values of x_1 , x_2 and x_3 .

$x_1 = \dots\dots\dots$

$x_2 = \dots\dots\dots$

$x_3 = \dots\dots\dots$

(3)

(b) Explain the relationship between the values of x_1 , x_2 and x_3 and the equation $x^3 - 2x^2 - 5 = 0$

.....

.....

.....

(2)

(Total for question 13 is 5 marks)

14 Using $x_{n+1} = 1 + \frac{1}{x_n^2}$

With $x_0 = 2$

(a) Find the values of x_1, x_2 and x_3 .

$x_1 = \dots\dots\dots$

$x_2 = \dots\dots\dots$

$x_3 = \dots\dots\dots$

(3)

(b) Explain the relationship between the values of x_1, x_2 and x_3 and the equation $x^3 - x^2 - 1 = 0$

.....

.....

.....

(2)

(Total for question 14 is 5 marks)