1 **a** $x = 0 \Rightarrow t = 2$

$x = 2 \Rightarrow t = 3$

b area $= \int_0^2 y \, dx$

$x = 2t - 4 \quad \therefore \frac{dx}{dt} = 2$

$\therefore$ area $= \int_2^3 \frac{1}{t} \times 2 \, dt$

$= \int_2^3 \frac{2}{t} \, dt$

c $= [2 \ln|t|]_2^3$

$= 2 \ln 3 - 2 \ln 2$

$= 2 \ln \frac{3}{2}$

d $t = \frac{x+4}{2}$

$\therefore y = \frac{2}{x+4}$

$\therefore$ area $= \int_0^2 \frac{2}{x+4} \, dx$

$= [2 \ln|x+4|]_0^2$

$= 2 \ln 6 - 2 \ln 4$

$= 2 \ln \frac{3}{2}$

2 **a** $x = 0 \Rightarrow \cos\theta = 0 \Rightarrow \theta = \frac{\pi}{2}, \frac{3\pi}{2}$

for $y > 0$, $\theta = \frac{\pi}{2}$ at A

$y = 0 \Rightarrow \sin\theta = 0 \Rightarrow \theta = 0, \pi$

for $x > 0$, $\theta = 0$ at B

b $x = 4\cos\theta \quad \therefore \frac{dx}{d\theta} = -4\sin\theta$

$\therefore$ area $= \int_{\frac{\pi}{2}}^{0} 2\sin\theta \times -4\sin\theta \, d\theta$

$= \int_0^{\frac{\pi}{2}} 8\sin^2\theta \, d\theta$

c shaded area $= \int_0^{\frac{\pi}{2}} (4 - 4\cos 2\theta) \, d\theta$

$= [4\theta - 2\sin 2\theta]_0^{\frac{\pi}{2}}$

$= (2\pi - 0) - (0 - 0)$

$= 2\pi$

area of ellipse $= 4 \times 2\pi$

$= 8\pi$

3 **a** $y = 0 \Rightarrow \sin 2t = 0 \Rightarrow t = 0, \frac{\pi}{2}$

$x = 2\sin t \quad \therefore \frac{dx}{dt} = 2\cos t$

area above x-axis

$= \int_0^{\frac{\pi}{2}} 5\sin 2t \times 2\cos t \, dt$

$= \int_0^{\frac{\pi}{2}} 10\sin 2t \cos t \, dt$

area enclosed by curve

$= 2\int_0^{\frac{\pi}{2}} 10\sin 2t \cos t \, dt$

$= \int_0^{\frac{\pi}{2}} 20\sin 2t \cos t \, dt$

b $= 40\int_0^{\frac{\pi}{2}} \sin t \cos^2 t \, dt$

$= -40\int_0^{\frac{\pi}{2}} (-\sin t)\cos^2 t \, dt$

$= -40[\frac{1}{3}\cos^3 t]_0^{\frac{\pi}{2}}$

$= -\frac{40}{3}(0 - 1)$

$= 13\frac{1}{3}$