

Name: _____
Date: _____

PROOF OF CIRCLE THEOREMS

GCSE
Edexcel
Mathematics
Grade 8/9

Mark	Score (%)
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Materials

For this paper you must have:

- Ruler
- Pencil, Rubber, Protractor and Compass
- Scientific calculator, which you are expected to use when appropriate

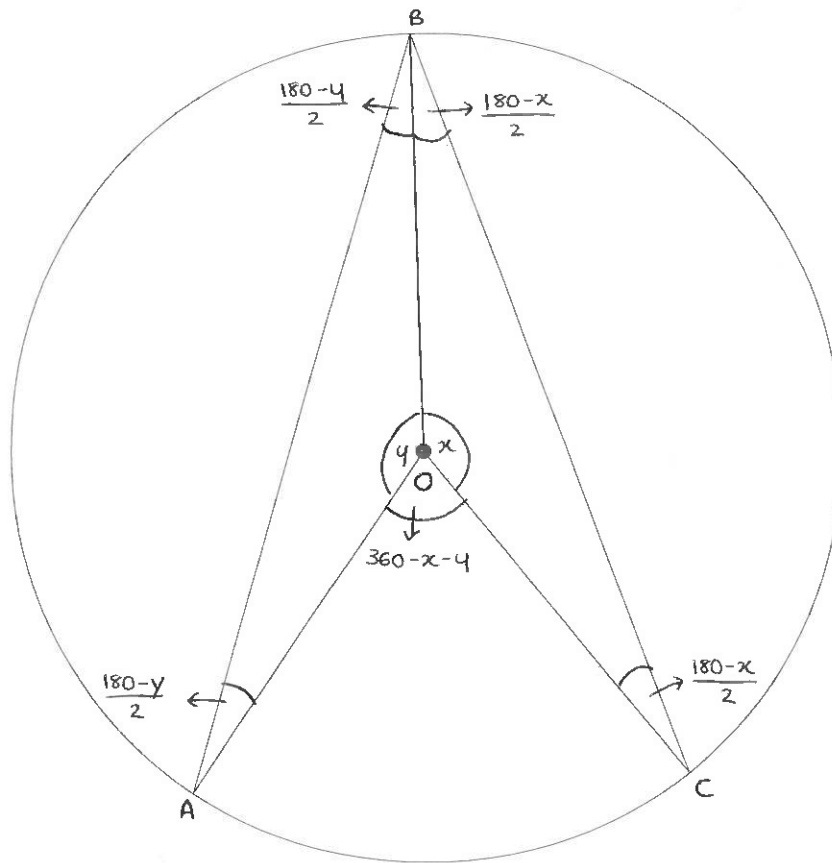
Instructions

- Answer all questions
- Answer questions in the space provided
- All working must be shown
- Do all rough work in this book. Cross out any rough work you don't want to be marked

Information

- The marks for the questions are shown in brackets

1



Prove that the angle subtended by an arc at the centre of a circle is twice the angle subtended at any point on the circumference.

$$\text{Angle } BOC = x$$

$$\text{Angle } BOA = y$$

$$\therefore \text{Angle } AOC = 360 - x - y$$

2 angles in isosceles triangle are the same

$$\therefore \text{Angles } OBC \text{ and } OCB = \frac{180 - x}{2}$$

$$\therefore \text{Angles } OBA \text{ and } OAB = \frac{180 - y}{2}$$

$$\begin{aligned} \text{Angle } ABC &= \frac{180 - x}{2} + \frac{180 - y}{2} \\ &= 90 - \frac{1}{2}x + 90 - \frac{1}{2}y \\ &= 180 - \frac{1}{2}x - \frac{1}{2}y \end{aligned}$$

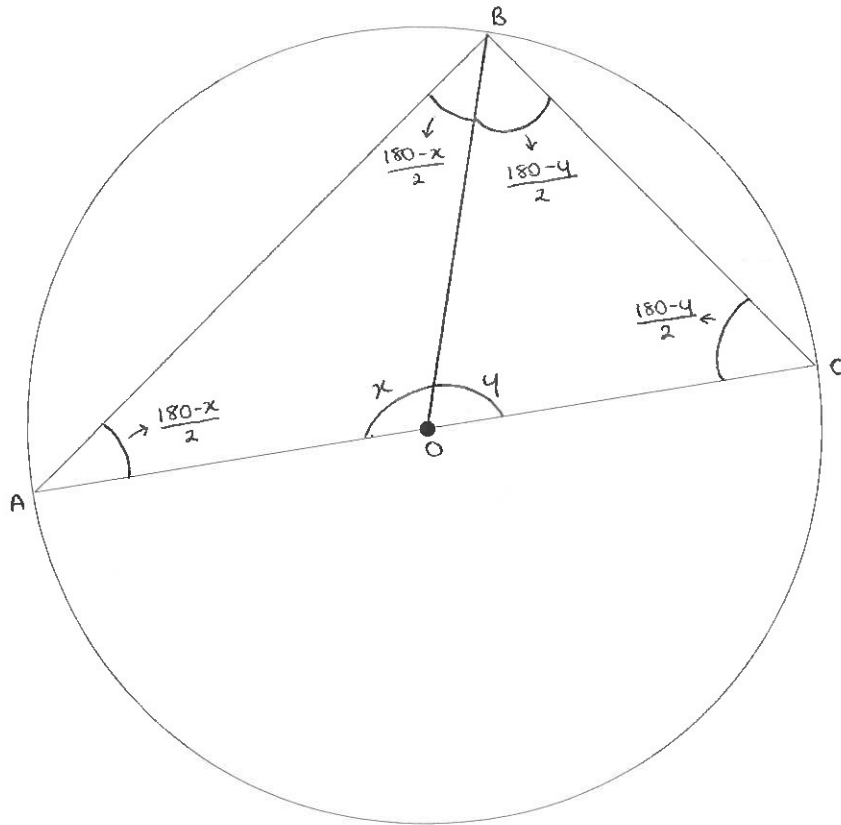
$$360 - x - y = 2 \left(180 - \frac{1}{2}x - \frac{1}{2}y \right)$$

$$360 - x - y = 2 \left(180 - \frac{1}{2}x - \frac{1}{2}y \right)$$

(Total for question 1 is 4 marks)

2

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Prove the angle subtended at the circumference by a semicircle is a right angle.

$$\text{Angle } AOB = x$$

$$\text{Angle } COB = y$$

$$\therefore x + y = 180^\circ \Rightarrow \text{angles in a straight line add up to } 180^\circ$$

2 angles in an isosceles triangle are the same

$$\therefore \text{Angles } ABO \text{ and } BAO = \frac{180-x}{2}$$

$$\therefore \text{Angles } CBO \text{ and } BCO = \frac{180-y}{2}$$

$$\text{Angle } ABC = \frac{180-x}{2} + \frac{180-y}{2}$$

$$= 90 - \frac{1}{2}x + 90 - \frac{1}{2}y$$

$$= 180 - \frac{1}{2}x - \frac{1}{2}y$$

$$\text{If } x + y = 180^\circ, \text{ then } \frac{1}{2}x + \frac{1}{2}y = 90^\circ$$

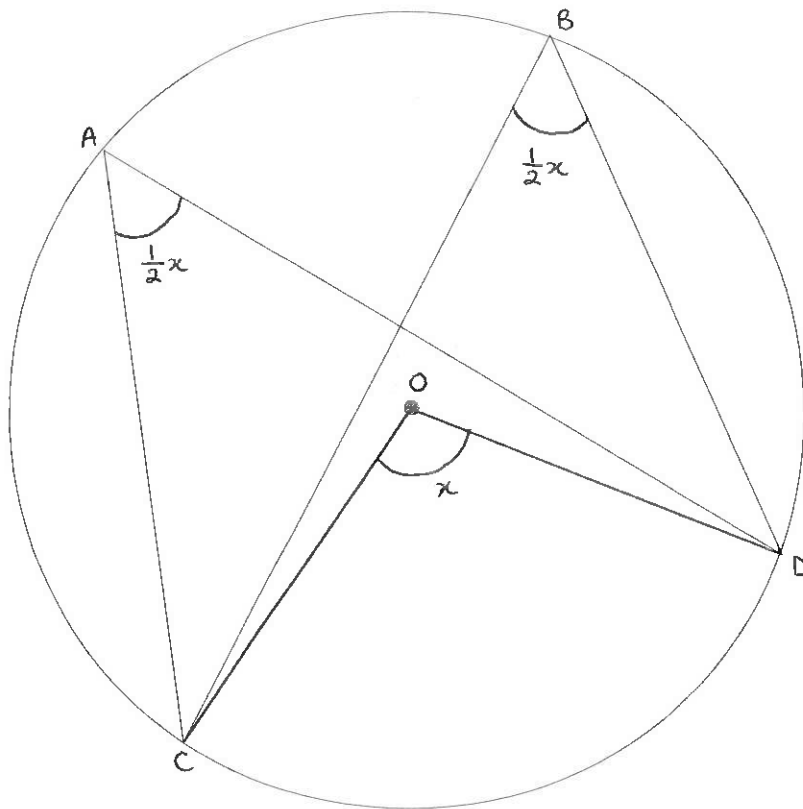
$$\therefore \text{Angle } ABC = 180 - \left(\frac{1}{2}x + \frac{1}{2}y\right)$$

$$= 180 - 90$$

$$= 90^\circ$$

(Total for question 2 is 4 marks)

3

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Prove that angles in the same segment are equal.

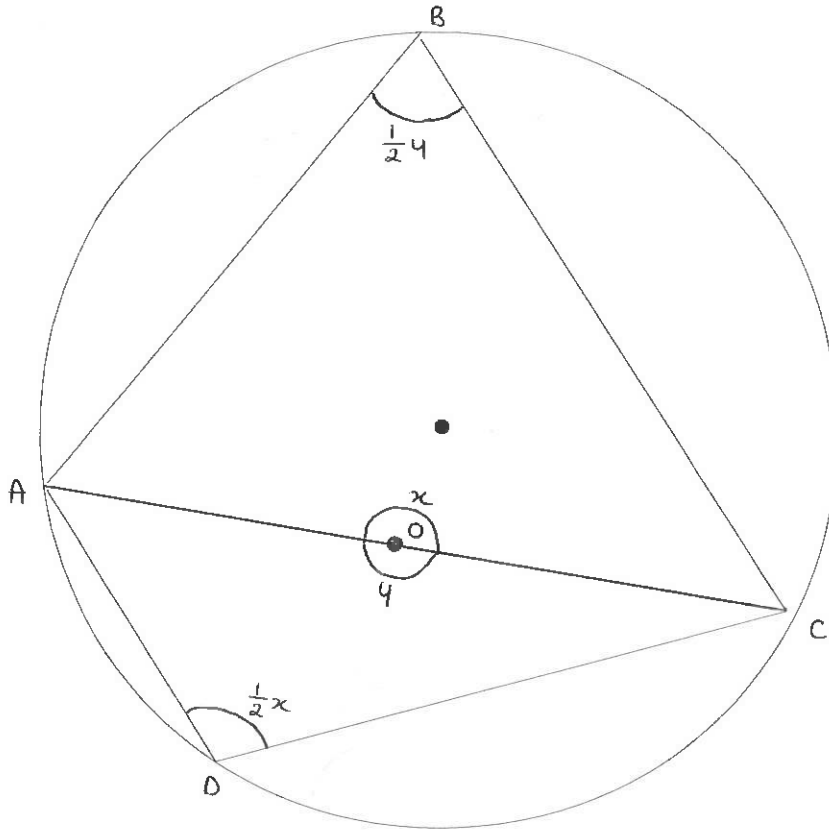
$$\text{Angle } COD = x$$

Angles CAD and CBD = $\frac{1}{2}x \Rightarrow$ angles at circumference are half of the angle at the centre

$$\therefore \frac{1}{2}x = \frac{1}{2}x$$

(Total for question 3 is 4 marks)

4



Prove that opposite angles of a cyclic quadrilateral sum to 180°

let angle AOC (top) = x

let angle AOC (bottom) = y

Angle $x + y = 360^\circ \Rightarrow$ angle at a point is 360°

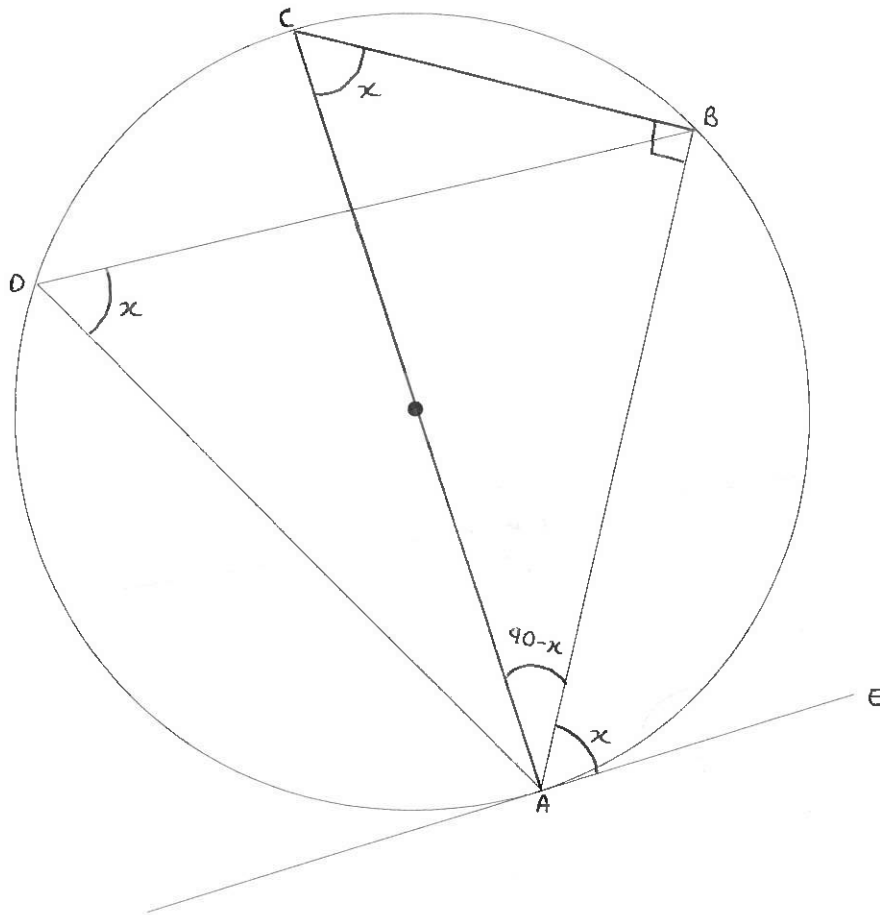
Angle ADC = $\frac{1}{2}x$

and Angle ABC = $\frac{1}{2}y \Rightarrow$ angles at a circumference is half of the angle at the centre

If $x + y = 360^\circ$, then $\frac{1}{2}x + \frac{1}{2}y = 180^\circ$

(Total for question 4 is 4 marks)

5

Leave
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Prove the alternate segment theorem.

Let Angle $BAE = x$

\therefore Angle $BAC = 90 - x \Rightarrow$ when tangent meets radius is 90°

\therefore Angle $ACB = x \Rightarrow$ angles in a triangle add up to 180°

$$\therefore 180 - 90 - (90 - x) = 90 - 90 + x = x$$

\therefore Angle $ADB = x \Rightarrow$ angles in same segment are equal

.....
(Total for question 10 is 2 marks)

